

QUIZ 4

Problem 1. Solve the boundary value problem

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial v}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 v}{\partial \theta^2} = 0 \quad 0 < r < c, -\pi \leq \theta \leq \pi,$$

$$v(c, \theta) = \cos^2(\theta).$$

Problem 2. Solve the boundary value problem

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial v}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 v}{\partial \theta^2} = 0 \quad 0 < r < c, -\pi \leq \theta \leq \pi,$$

$$v_r(c, \theta) = \sin^2(\theta).$$

Problem 3. Solve the boundary value problem

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial v}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 v}{\partial \theta^2} = 0 \quad 1 < r < 2, -\pi \leq \theta \leq \pi,$$

$$v(1, \theta) = \cos^2(\theta), \quad v(2, \theta) = \frac{1}{8}(\cos^2\theta + 1).$$

Problem 4. Solve the boundary value problem

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial v}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 v}{\partial \theta^2} = 0 \quad 0 < r < c, -\pi \leq \theta \leq \pi,$$

$$v(c, \theta) = \begin{cases} 1 & \text{if } 0 \leq \theta < \pi, \\ 0 & \text{if } -\pi < \theta \leq 0. \end{cases}$$