

## PRACTICE EXAM 4

**Problem 1.** Given two matrices

$$A = \begin{pmatrix} 1 & 2 & 0 \\ -1 & -4 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 1 & 0 \\ -1 & -1 & 0 \\ 1 & 1 & 0 \end{pmatrix}.$$

Find the matrices

$$2A - B, \quad AB, \quad B(A + B).$$

**Problem 2.** Find the general solution to the system of linear ordinary differential equations

$$\frac{dx}{dt} = Ax, \quad A = \begin{pmatrix} 3 & -2 \\ 4 & -1 \end{pmatrix}$$

*Answer.*

$$x(t) = C_1 e^t \begin{pmatrix} \cos(2t) \\ \cos(2t) + \sin(2t) \end{pmatrix} + C_2 e^t \begin{pmatrix} \sin(2t) \\ \sin(2t) - \cos(2t) \end{pmatrix}.$$

**Problem 3.** Solve the initial value problem

$$\frac{dx}{dt} = Ax, \quad x(0) = (1, 1) \quad A = \begin{pmatrix} 1 & 1 \\ 2 & 0 \end{pmatrix}$$

*Answer.*

$$x(t) = e^{2t} \begin{pmatrix} 1 \\ 1 \end{pmatrix}.$$

Typeset by  $\mathcal{A}\mathcal{M}\mathcal{S}\text{-}\mathcal{T}\mathcal{E}\mathcal{X}$

**Problem 4.** Find the general solution to the system of linear ordinary differential equations

$$\frac{dx}{dt} = Ax, \quad A = \begin{pmatrix} 0 & 1 & -1 \\ 1 & 0 & -1 \\ 2 & 2 & -3 \end{pmatrix}$$

*Answer.*

$$x(t) = C_1 e^{-t} \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} + C_2 e^{-t} \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} + C_3 e^{-t} \begin{pmatrix} t+1 \\ t \\ 2t \end{pmatrix}.$$