

PRACTICE EXAM (FINAL)

Problem 1.

Solve the boundary value problem

$$u_{xx} + u_{yy} = 0 \quad 0 \leq x \leq \pi, \quad 0 \leq y \leq \pi,$$

$$u_x(0, y) = u_x(\pi, y) = 0$$

$$u(x, 0) = \cos^2(x), \quad u(x, \pi) = 1.$$

Problem 2.

Solve the boundary value problem

$$u_t - u_{xx} + u = 0 \quad 0 \leq x \leq 1$$

$$u(0, t) = u(1, t) = 0$$

$$u(x, 0) = \sin(\pi x)$$

Problem 3.

Solve the boundary value problem

$$u_{xx} + u_{yy} = 0 \quad \text{for all } (x, y) \text{ such that } x^2 + y^2 \geq 1,$$

$$u(x, y) = \cos^3(\theta) \text{ for } x^2 + y^2 = 1.$$

Problem 4.

Solve the boundary value problem

$$u_{tt} - u_{xx} + u = 0 \quad 0 \leq x \leq 1$$

$$u(0, t) = u(1, t) = 0$$

$$u(x, 0) = \sin(\pi x), \quad u_t(x, 0) = 0$$

Problem 5

Let $u(x, t)$ be a solution to the boundary value problem

$$u_{tt} = u_{xx} \quad -\infty < x < \infty$$

$$u(x, 0) = e^{-x^2}, \quad u_t(x, 0) = 0$$

Find $u(1, 1)$