

PRACTICE EXAM 2 (267 FALL 2003)

Problem 1. Find at least one solution to the endpoint problem

$$y'' + 4y = 1, y(\pi) = 0, y(0) = 0.$$

Answer.

$$y(t) = -\frac{\cos(2t)}{4} + \frac{1}{4} + C_1 \sin(2t).$$

Problem 2. Find a general solution to the ordinary differential equation

$$y^{(3)} + y'' + y' = 1.$$

Answer.

$$y(t) = t + C_1 + C_2 e^{-\frac{t}{2}} \cos\left(\frac{\sqrt{3}t}{2}\right) + C_3 e^{-\frac{t}{2}} \sin\left(\frac{\sqrt{3}t}{2}\right).$$

Problem 3. Solve the initial value problem

$$y^{(4)} + y'' = 12\cos(2t), \quad y(0) = 1, y'(0) = y''(0) = y^{(3)}(0) = 0.$$

Answer

$$y(t) = 4 - 4\cos(t) + \cos(2t).$$

Problem 4. Find a general solution to the ordinary differential equation

$$y'' + y = 2\sec^3 t.$$

Answer.

$$y(t) = c_1 \cos t + c_2 \sin t - \frac{\cos(2t)}{\cos t}.$$