IMPORTANT FORMULAS FOR THE CHAPTER 12.

1. A surface given as a graph of a function $z = f(x, y)$. Let the point $P = (x_0, y_0, z_0)$ be on this surface.

An equation of the tangent plane to the surface $z = f(x, y)$ *at the point* $P = (x_0, y_0, z_0)$ *is*

$$z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0).$$

A surface given by the equation $F(x, y, z) = 0$. Let the point $P = (x_0, y_0, z_0)$ be on this surface. *An equation of the tangent plane to the surface* $F(x, y, z) = 0$ *at the point* $P = (x_0, y_0, z_0)$ *is*

$$F_x(x_0, y_0, z_0)(x - x_0) + F_y(x_0, y_0, z_0)(y - y_0) + F_z(x_0, y_0, z_0)(z - z_0) = 0.$$

Total differential

Suppose we have a function $z = f(x, y)$. Assume that this function has continuous partial derivatives $f_x(x, y), f_y(x, y)$.

The total differential of the function $z = f(x, y)$ *is given by formula*

$$dz = f_x(x, y)dx + f_y(x, y)dy.$$

Let $w = f(x, y, z)$ be a function of three variables with continuous partial derivatives $f_x(x, y), f_y(x, y)$ and $f_z(x, y)$. *The total differential of the function* $w = f(x, y, z)$ *is given by formula*

$$dw = f_x(x, y, z)dx + f_y(x, y, z)dy + f_z(x, y, z)dz.$$

Chain Rules

Chain Rule 1. Let function $z = f(x, y)$ has the first order continuous derivatives. Assume $x = g(t)$ and $y = h(t)$. Let functions $g(t)$ and $h(t)$ be differentiable. We consider the composite function $z = f(g(t), h(t))$. Then

$$\frac{dz}{dt} = f_x(g(t), h(t))g'(t) + f_y(g(t), h(t))h'(t).$$

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Chain Rule 2. Let function $z = f(x, y)$ has the first order continuous derivatives. Assume $x = g(s, t)$ and $y = h(s, t)$. Let functions $g(s, t)$ and $h(s, t)$ be differentiable. We consider the composite function $z = f(g(s, t), h(s, t))$. Then

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\frac{\partial g}{\partial s} + \frac{\partial z}{\partial y}\frac{\partial h}{\partial s}$$
$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\frac{\partial g}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial h}{\partial t}$$

Directional Derivative

Let $\mathbf{u} = (a, b)$ be a unit vector ($|\mathbf{u}| = 1$.) Suppose we have a function $z = f(x, y)$ with continuous partial derivatives.

The directional derivative of the function f at direction of unit vector $\mathbf{u}$ given by formula

$$D_u f(x, y) = f_x(x, y)a + f_y(x, y)b.$$

The gradient of f is the vector function ∇f defined by formula

$$\nabla f(x, y) = (f_x(x, y), f_y(x, y)).$$

Maximum/Minimum Values

Let a function $f(x, y)$ be a smooth function. Suppose that $(\hat{x}, \hat{y})$ is the point of local maximum or minimum of this function. Then

$$f_x(\hat{x}, \hat{y}) = 0, \quad f_y(\hat{x}, \hat{y}) = 0.$$

Second Derivatives Test. Suppose the second partial derivatives of the function f are continuous in a disk with center (a, b) and suppose that $\nabla f(a, b) = 0$. Let

$$D = f_{xx}(a, b)f_{yy}(a, b) - (f_{xy}(a, b))^2$$

a) If $D > 0$ and $f_{xx}(a, b) > 0$ then (a, b) is a point of local minimum.
b) If $D > 0$ and $f_{xx}(a, b) < 0$ then (a, b) is a point of local maximum.
c)If $D < 0$ then (a, b) is not local maximum or minimum.

Lagrange Multipliers. Suppose we are looking for a maximum(minimum) of the function $f(x, y, z)$ over all (x, y, z) such that $g(x, y, z) = k$. Here k is the given number. In order to find a point of maximum (minimum) we are supposed to solve the system of equations

$$\begin{cases} \nabla f(x, y, z) = \lambda \nabla g(x, y, z) \\ g(x, y, z) = k. \end{cases}$$

We have four equations here and for unknowns (x, y, z, λ). (λ is called the Lagrangian multiplier.)