

HOMEWORK 1(267, SPRING 2005)

Problem 1. Let $y(t)$ be solution to the initial value problem

$$\frac{dy}{dt} = \frac{2ty}{y^2 - t^2}, \quad y(1) = 3^{\frac{1}{2}},$$

Find $y(3)$

A) $y(3) = \frac{2}{4}$; **B)** $y(3) = \frac{1}{2}$; **C)** $y(3) = \frac{4}{e}$; **D)** $y(3) = \frac{\pi}{e}$; **E)** $y(3) = 5\sqrt{3}$; **F)** none of above.

Problem 2. $y(x)$ be solution to the initial value problem

$$\frac{dy}{dx} + y = x^3y, \quad y(0) = \sqrt{2},$$

Find $y(\frac{1}{2})$

A) $y(\frac{1}{2}) = 3$; **B)** $y(\frac{1}{2}) = \frac{1}{3}$; **C)** $y(\frac{1}{2}) = 1$; **D)** $y(\frac{1}{2}) = \sqrt{2}$; **E)** $y(\frac{1}{2}) = -2$; **F)** none of above.

Problem 3. $y(x)$ be solution to the initial value problem

$$y'' - 2y' + y = 0, \quad y(2) = 1, \quad y'(2) = -2$$

Find $y(4)$

A) $y(4) = -5$; **B)** $y(4) = \frac{1}{3}e^2$; **C)** $y(4) = 1$; **D)** $y(4) = -5e^2$; **E)** $y(4) = -3e^4$; **F)** none of above.

Problem 4. Let $y(t)$ be solution to the initial value problem

$$\frac{dy}{dt} = \frac{3t^2 + 4t + 2}{2(y - 1)}, \quad y(0) = -1$$

Then

A) $y(1) = -2.$

B) $y(1) = -3.$

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C) $y(1) = -4.$

D) $y(1) = \sqrt{2}.$

F) None of above.

Problem 5. Let N_0 be some number such that $0 < N_0 < 1$. Let $N(t)$ be a solution to the initial value problem

$$\frac{dN}{dt} = N^2(N^2 - 1), \quad N(0) = N_0, \quad t \geq 0.$$

Denote by $I = [0, A)$ the maximal interval of existence of a solution to this problem. Then

A) $A = 2\sqrt{N_0}.$

B) $A = \frac{1}{N_0}.$

C) $A = \frac{1}{1 + N_0^3}.$

D) $A = +\infty.$

F) none of above.