FINAL EXAM

Fall 267

Circle the correct answer. **If you circle more than one answer the credit for a problem is zero.**

Problem 1.(*10 POINTS.)*Solution $y(x)$ to the initial value problem

$$(x+2xy)\frac{dy}{dx} = -(x^2+y^2+y), \quad y(1)=1$$

satisfies the equation:

A) $y^2x + y(x)x = \dfrac{x^3}{3} - \dfrac{7}{3}$

B) $y(x) = \dfrac{1}{x} + sin(x-1)$

C) $y(x) = \sqrt{\dfrac{x+1}{(x-1)^2+2}}$

D) $y^2(x)x + y(x)x = -\dfrac{x^3}{3} + \dfrac{7}{3}$

E) $3y(x) + x^2 + y^2(x) + y(x) = 6$

F) Non one of above.

Problem 2.(*15 POINTS.)* Let $y(x)$ be solution to the initial value problem

$$y'' + 4y' + 4y = cos(x) + 5\delta(x-4\pi), \quad y(0)=0, \quad y'(0) = \frac{1}{4}.$$

Find $y(2\pi)$.

A) $y(2\pi) = 0$; **B)** $y(2\pi) = -1$; **C)** $y(2\pi) = 1$; **D)** $y(2\pi) = 2$; **E)** $y(2\pi) = -2$; **F)** $y(2\pi) = \frac{3}{4}$; **J)** $y(2\pi) = 8$; **L)** Non one of above.

Problem 3.(*10 POINTS.)* Let $y(x)$ be solution to the initial value problem

$$y'' + 4y = cos(x) + 5u(x - 2\pi), \quad y(0) = 1, y'(0) = 1.$$

Find $y(\pi)$.

A) $y(\pi) = 0$; **B)** $y(\pi) = -1$; **C)** $y(\pi) = 1$; **D)** $y(\pi) = 2$; **E)** $y(\pi) = -2$; **F)** $y(\pi) = \frac{3}{4}$; **J)** $y(\pi) = 8$; **L)** Non one of above.

Problem 4. (*15 POINTS.)* Let vector function $X(t) = (x_1(t), x_2(t))$ be solution to the initial value problem

$$X' = \begin{pmatrix} 2 & -1 \\ -2 & 1 \end{pmatrix} X + \begin{pmatrix} 0 \\ 18t \end{pmatrix}, \quad X(0) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

Find $X(1)$.

A) $X(1) = \begin{pmatrix} e^3 - 5 \\ e^3 + 2 \end{pmatrix}$

B) $X(1) = \begin{pmatrix} 4e^3 - 1 \\ 3e^3 + 1 \end{pmatrix}$

C) $X(1) = \begin{pmatrix} 7e^3 - 4 \\ 3e^3 - 2 \end{pmatrix}$

D) $X(1) = \begin{pmatrix} 2e^3 - 2 \\ e^3 + 6 \end{pmatrix}$

E) $X(1) = \begin{pmatrix} e^3 \\ e^3 + 3 \end{pmatrix}$

F) $X(1) = \begin{pmatrix} -e^3 - 5 \\ e^3 + 2 \end{pmatrix}$

G) $X(1) = \begin{pmatrix} -e^3 + 6 \\ e^3 + 4 \end{pmatrix}$

H) $X(1) = \begin{pmatrix} 2e^3 + 3 \\ e^3 + 2 \end{pmatrix}$

L) Non one of above.

Problem 5.(*15 POINTS.*) Let vector function $X(t) = (x_1(t), x_2(t))$ be solution to the initial value problem

$$X' = \begin{pmatrix} 5 & 3 \\ -3 & -1 \end{pmatrix} X, \quad X(0) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

Find $X(1)$

A) $X(1) = e^2 \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

B) $X(1) = e^2 \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

C) $X(1) = e^2 \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

D) $X(1) = e^2 \begin{pmatrix} 4 \\ 7 \end{pmatrix}$

E) $X(1) = e^2 \begin{pmatrix} 2 \\ \frac{4}{3} \end{pmatrix}$

F) $X(1) = e^2 \begin{pmatrix} 3 \\ -1 \end{pmatrix}$

G) $X(1) = e^2 \begin{pmatrix} 3 \\ -2 \end{pmatrix}$

H) $X(1) = e^2 \begin{pmatrix} 3 \\ -4 \end{pmatrix}$

L) Non one of above.

Problem 6.(*10 POINTS.*) Let function $y(x)$ be solution to the initial value problem

$$\frac{dy}{dx} = xy^2, \quad y(0) = 1.$$

Find $y(1)$.

A) $y(1) = -1$, **B)** $y(1) = -2$, **C)** $y(1) = 0$, **D)** $y(1) = 2$, **E)** $y(1) = -3$, **F)** $y(1) = 3$, **G)** $y(1) = 4$, **H)** $y(1) = 5$. **L)** Non one of above.

Problem 7. (*15 POINTS.)*
Let function $y(x)$ be solution to the initial value problem

$$y'' - 2y' + y = x^2 + 2cos(x), \quad y(0) = 1, \quad y'(0) = 0.$$

Find $y(\pi)$.

A) $y(\pi) = \pi^2 + 8\pi + 13 - \frac{5}{2}e^{\pi}$.
B)$y(\pi) = \pi^2 + 11\pi + 14 - \frac{11}{2}e^{\pi} + 6\pi e^{\pi}$.
C) $y(\pi) = 2\pi^2 + 6\pi + 13 - \frac{7}{2}e^{\pi} + 3\pi e^{\pi}$.
D) $y(\pi) = 2\pi^2 + 3\pi + 12 - \frac{7}{2}e^{\pi} + 2\pi e^{\pi}$.
E) $y(\pi) = 2\pi^2 + 5\pi + 14 - 6e^{\pi} + 6\pi e^{\pi}$.
F) $y(\pi) = 3\pi^2 + 7\pi + 13 - 7e^{\pi} + 9\pi e^{\pi}$.
G) $y(\pi) = -\pi^2 + 7\pi + 12 - 9e^{\pi} + 14\pi e^{\pi}$.
H) $y(\pi) = \pi^2 + 4\pi + 14 - 13e^{\pi} + 11\pi e^{\pi}$.
J) $y(\pi) = \pi^2 + \pi + 12 - 15e^{\pi} + 10\pi e^{\pi}$.
L) Non one of above.

Problem 8. (*10 POINTS.)* General solution to the ordinary differential equation

$$y'' + 2y' + y = x + e^x + cos(x).$$

given by the formula

A) $y(x) = C_1e^{-x} + C_2e{-}x + x - 1 + cos(x) + sin(x) + \frac{1}{4}e^x$

B) $y(x) = C_1e^{-x} + C_2xe^{-x} + x - 2 + cos(x) - sin(x) + \frac{1}{4}e^x$

C) $y(x) = C_1e^{-x} + C_2xe^{-x} + x - 1 + cos(x) + \frac{1}{4}e^x$

D) $y(x) = C_1e^{-x} + C_2x - 2 + \frac{1}{2}sin(x) + \frac{1}{4}e^x$

E) $y(x) = C_1e^{-x} + C_2xe^{-x} + x - 2 + \frac{1}{2}cos(x) + \frac{1}{4}e^x$

F) $y(x) = C_1e^{-x} + C_2xe^{-x} + x - 2 + \frac{1}{2}sin(x) + \frac{1}{4}e^x$

G) $y(x) = C_1e^{-x} + C_2xe^{-x} + x - 2 + \frac{1}{2}sin(x) - \frac{1}{4}e^x$

L) Non one of above.