## ANSWERS FOR EXAM 1.

**Problem 1.** We should minimize the function $f(x, y, z) = x^2 + y^2 + z^2$ under the constrain equation $h(x, y, z) = x + y + z - 1 = 0$. Note that

$$\nabla f(x, y, z) = (2x, 2y, 2z), \quad \nabla h(x, y, z) = (1, 1, 1).$$

By the method of Lagrange multipliers

$$\begin{cases} 2x = \lambda \\ 2y = \lambda \\ 2z = \lambda \\ x + y + z = 1 \end{cases}$$

From the first three equations we have

$$x = y = z.$$

So we can rewrite the fourth equation as

$$3x = 1$$

Hence the closest point is $(\frac{1}{3}, \frac{1}{3}\frac{1}{3})$.

**Problem 2.** First we need the partial derivatives for the function $f(x, y, z)$

$$f_x(x, y, z) = cos(x^2+y^2+z^2) - 2xsin(x^2+y^2+z^2) + 1, \quad f_y(x, y, z) = -2xysin(x^2+y^2+z^2) + 1,$$
$$f_z(x, y, z) = -2xzsin(x^2 + y^2 + z^2).$$

Then $f_x(0, 0, 0) = 2$, $f_y(0, 0, 0) = 1$, $f_z(0, 0, 0) = 0$. Using the formula for the total differential we have

$$dw = 2dx + dy.$$

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**Problem 3.** Let $P = (x_0, y_0, z_0)$ be a point on the paraboloid $z = x^2 + (y-1)^2$. Then the tangent plane to the paraboloid at this point given by formula

$$2x_0(x - x_0) + 2(y_0 - 1)(y - y_0) - (z - z_0) = 0.$$

This plane is parallel to the plane $z = -1$ if and only if there exists $\lambda > 0$ such that

$$\begin{cases} 2\lambda x_0 = 0 \\ 2\lambda(y_0 - 1) = 0 - \lambda = 1 \end{cases}$$

from the third equation we have $\lambda = -1$. We can plug in this value of $\lambda$ into first two equations

$$-2x_0 = 0, \quad -2(y_0 - 1) = 0.$$

Hence $x_0 = 0$,$y_0 = 1$. Since the point is assumed to be on the paraboloid we have $z_0 = x_0^2 + (y_0 - 1)^2$. Hence $z_0 = 0$.

**Problem 4.** Let find the first partial derivatives of the function $f(x, y)$

$$f_x(x, y) = 2x - 10y + 2 \quad f_y(x, y) = -10x + 2y$$

Solving the linear system

$$\begin{cases} x - 5y = -1 \\ -5x + y = 0 \end{cases}$$

we have $y = \frac{5}{24}$ and $x = \frac{1}{24}$. Since $f_{xx}(x, y) = 2, f_{yy}(x, y) = 2$ and $f_{xy}(x, y) = -10$ we have

$$\mathcal{D} = 2 \cdot 2 - 10^2 < 0$$

So the point $(\frac{1}{24}, \frac{5}{24})$ is the saddle point.