Instructions: Read the question carefully and answer all parts of the question. At the end of the test look to see if you answered all parts of the question. Ask if something is unclear.

1) (15 pt) Convert the following system of equations into the form \( \frac{dx}{dt} = A(t)x + f(t) \). (Give the meaning of \( A \), \( x \), and \( f \), but do not solve.)

\[
x'' + x' + t^2 y = \cos t, \quad y' + x = \sin t.
\]

2) (20 pt) Find a real-valued general solution to the following:

\[
x' = \begin{bmatrix} 7 & -4 \\ 4 & -1 \end{bmatrix} x.
\]

3) (20 pt) The following matrix \( A \) has eigenvalues \(-2, -2, 5\). Find the general solution to \( x' = Ax \):

\[
A = \begin{bmatrix} -2 & 0 & 1 \\ -7 & 5 & 7 \\ 0 & 0 & -2 \end{bmatrix}
\]

4) (25 pt) Solve:

\[
x' = \begin{bmatrix} -3 & 2 \\ -3 & 4 \end{bmatrix}; \quad x(0) = \begin{bmatrix} 3 \\ 4 \end{bmatrix}.
\]

5) (20 pt) Use the matrix exponential \( e^{At} \) to find the solution to

\[
x' = \begin{bmatrix} -2 & 0 & 1 \\ 0 & -2 & 0 \\ 0 & 0 & 2 \end{bmatrix}; \quad x(0) = \begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix}
\]