\[
\frac{dx}{dt} = -x + \sin y \\
\frac{dy}{dt} = 2x
\]

Critical points: 
\[-x + \sin y = 0 \Rightarrow x = 0 \text{ and } y = k\pi\ k = 0, \pm 1, \pm 2, \ldots\]
\[
2x = 0
\]
\[
\Rightarrow (0, 0), (0, \pm \pi), (0, \pm 2\pi), \ldots \text{ are the crit. pts.}
\]
The graphs indicate 2 different types of crit. pts:
\[
\text{at } (0, \pm k\pi) \text{ } k \text{ odd: appears to be a stable spiral point (see p. 500)}
\]
\[
\text{at } (0, \pm k\pi) \text{ } k \text{ even: appears to be an unstable saddle point (see top p. 510)}
\]

Analyse on of each type:

Analysis at (0, 0)

1st linearize: \( (\sin y)'(0) = 1 \Rightarrow \sin y \approx 1 \cdot y \) near \( y = 0 \)

\[
\Rightarrow \frac{dx}{dt} = -x + y \\
\frac{dy}{dt} = 2x
\]

Stability analysis

\[
|A - \lambda I| = \lambda^2 + \lambda - 2 = (\lambda + 2)(\lambda - 1)
\]
\[
\lambda = -2 > 1 \Rightarrow \text{ unstable saddle point}
\]

Analysis of (0, \pi):

1st linearize: let \( \tilde{y} = y - \pi \)

(\text{so that } \tilde{y} \text{ is near } 0)

then \[
\frac{d\tilde{y}}{dt} = \frac{d(y-\pi)}{dt} = \frac{dy}{dt} = 2x
\]

and \( \sin y = \sin(\tilde{y} + \pi) \approx [\sin(\tilde{y} + \pi)]'(0) \cdot \tilde{y} \)

so \[
\frac{dx}{dt} = -x + -\tilde{y}
\]

\[
A = \begin{bmatrix}
-1 & -1 \\
2 & 0
\end{bmatrix}
\]

\[
\Rightarrow \lambda^2 + \lambda + 2 = 0
\]
\[
\Rightarrow \lambda = -\frac{1}{2} \pm \frac{\sqrt{7}}{2} \Rightarrow \text{stable spiral pt.}
\]
\[ x' = -x + \sin(y) \]
\[ y' = 2x \]

Graph shows the equilibrium points \((0, -\pi), (0, 0), (0, \pi), (0, 2\pi), (0, 3\pi)\).
\[ x' = -x + \sin(y) \]
\[ y' = 2x \]

**stable spiral point at \((0, \pi)\)**
\[ x' = -x + \sin(y) \]
\[ y' = 2x \]

Unstable saddle at \((0,0)\)