1) Solve the following initial value problems by any valid method. Give the answer explicitly.
   a) (10pt) \( ty' + y = \sin t \quad y(\pi) = 1 \)

   b) (15pt) \( 2y'(y + x^4) + 8yx^3 + 8x^7 = 0 \quad y(0) = 1 \)

2) Find the general solution.
   a) (10pt) \( y'' - 6y' + 9y = 0 \)

   b) (10pt) \( y'' + 2y' + 5y = \cos 2t \)

3) Let \( A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \). Determine the following:
   a) (20pt) all eigenvalues and eigenvectors of \( A \):
   b) (5pt) is \( A \) singular or nonsingular?
   c) (5 pt) the general solution to \( \mathbf{x}' = A\mathbf{x} \)
4) Let \( A = \begin{pmatrix} 2 & 1 \\ 4 & -1 \end{pmatrix} \).

a) find the solution to \( x' = Ax \) with initial conditions: \( x_1(0) = 2, \ x_2(0) = 1 \).

b) Sketch the direction field and classify the origin as one of: stable node, unstable node, stable spiral point, unstable spiral point, saddle point. Draw in the solution for \( t \geq 0 \) that satisfies the initial conditions in part a).

5) (15pt) Find all equilibrium solutions for \( y' = x \sin x \) and use a phase line diagram to determine their stability properties.

6) (30pt) Find the two critical points of the following system which are the closest to the origin and determine their stability properties. Also indicate the type of critical point (stable node, saddle point, unstable spiral point, etc.)

\[
x' = \cos(x - y), \quad y' = x^2 - y^2.
\]