Math 266    Test 2    Spring 2004    Name:

Answer all parts of the following questions. All solutions to DE’s should be expressed in terms of real quantities whenever possible.

1) Let \( L[y] = (1 - x)y'' + xy' - y. \)
   (a) (3 pts) Is the DE \( L[y] = 0 \) linear or nonlinear?
   (b) (3 pts) Is the DE \( L[y] = 0 \) homogeneous or nonhomogeneous?
   (c) (4 pts) If the initial conditions \( y(0) = 2, \ y'(0) = 2 \) are specified, what is the interval of existence and uniqueness?
   (d) (10pts) Show that \( y_1(x) = x \) and \( y_2(x) = e^x \) form a fundamental set of solutions for the DE \( L[y] = 0 \). (i.e., show they solve the DE and satisfy the Wronskian condition.)
   (e) (10pts) use the variation of parameters formula to find the general solution to \( L[y] = e^x(1 - x) \)

2) Determine the general solution to the following:
   (a) (15pt) \( y'' + 2y' + y = 0, \ \ y(0) = 0, \ y'(0) = a \)
   (b) (10pt) \( y'' - 2y' + 5y = 0. \)
3) (15pt) A 10 lb force stretches a spring 1/4 foot. Assume that the spring also is attached to a viscous damper that exerts a force of 3 lb when the spring is stretched at a rate of 1 ft/sec. Suppose that a mass weighing 5 lb is attached to the end of the spring and is pulled down 3 inches below its equilibrium position and is released. Write down the initial value problem needed to determine the motion of the spring-mass system. (DO NOT SOLVE!)

4) (20pt) Find the solution: \( y'' + 2y' = 8t, \ y(0) = 1, \ y'(0) = 0. \)

5) (15pt) Write down the correct form of the particular solution \( Y \) of the following differential equation: \( y'' - 2y' + 2y = e^t + \sin t + e^t \sin t. \) (Do not solve for the constants.)

7) (5pt extra credit) Find the general solution: \( y'''' + y''' = 24x. \)