Math 166  Test 3  Solutions

Section:

1) (12pt) Let \( f(x) = \frac{x^2}{4 + x^2} \). Find the Maclaurin series for \( f(x) \). (This power series is somehow related to the one for \( \frac{1}{1 - x} \).)

Answer:

\[
\frac{x^2}{4} - \frac{x^4}{4^2} + \frac{x^6}{4^3} - \frac{x^8}{4^4} \ldots
\]

2) (12pt) Calculate the Taylor series in powers of \((x - 2)\) for \( f(x) = x^4 \). (Apply Taylor’s formula with \( a = 2 \) to \( f(x) \).)

Answer:

\[
16 + 32(x - 2) + 24(x - 2)^2 + 8(x - 2)^3 + (x - 2)^4
\]

Note that in this case \( f^{(n)}(2) = 0 \) for \( n \geq 5 \) so in this case we only get 5 nonzero terms, (not an infinite series).

3) (12pt) (a) Find the Maclaurin polynomial of order 3 for \( f(x) = x \tan^{-1} x \):

Ans: \( x(x - x^3/3 + x^5/5 + \ldots) \) to order 3 is just \( x^2 \). (To order 4 it would be \( x^2 - x^4/4 \).)

(b) Use this to approximate \( \int_{0}^{5/2} f(x) \, dx \).

Ans: \( \int_{0}^{5/2} x^2 \, dx = 1/24 \).

4) (12pt) (a) Sketch the curves \( r = \sin \theta, \quad r = \cos \theta \), labeling any \( x \) and \( y \) intercepts.

They are both circles of diameter 1 through the origin. One passes through (0,1) the other passes through (1,0).

(b) Set up the integral (or integrals) to find the area of the region inside both curves:

Ans: the circles intersect at \( \theta = \pi/4 \) and one way is

\[
\frac{1}{2} \int_{0}^{\pi/4} \sin^2 \theta \, d\theta + \frac{1}{2} \int_{\pi/4}^{\pi/2} \cos^2 \theta \, d\theta
\]

5) (12pt) Sketch the given curves (labeling all \( x \) and \( y \) intercepts) and find their points of intersection both in polar and cartesian \( r = 6, \quad r = 4 - 4 \cos \theta \)

This is a circle of radius 6 centered at the origin, and a cardioid (heart) positioned sideways, with the bottom at (-8,0). They intersect at angles \( \pm 2\pi/3 \)

Intersection(s) in polar \((\theta, r) = (\pm 2\pi/3, 6)\)

Intersection(s) in cartesian \((x, y) = (3, \pm 3\sqrt{3})\)