1) Let \( f(x) = x^2 \), \( g(x) = \sqrt{x} \) and let \( R \) be the region bounded by the graphs of \( y = f(x) \) and \( y = g(x) \). Express in terms of integrals (of the form \( \int_a^b h(x) \, dx \) or \( \int_c^d h(y) \, dy \)) each of the following quantities. Simplify the integrands but do not evaluate the integrals.

(a) (10pt) The moment of \( R \) with respect to the \( y \)-axis.
Solution: \( M_y = \int_0^1 x(\sqrt{x} - x^2) \, dx \)

(b) (10pt) The volume of the solid formed by rotating \( R \) about the \( y \)-axis.
Solution: Shell method gives: \( V = \int_0^1 2\pi x(\sqrt{x} - x^2) \, dx \)
Washer method gives \( V = \int_0^1 \pi(R^2 - r^2) \, dy = \int_0^1 \pi(y - y^4) \, dy \)

(c) (10pt) The volume of the solid formed by rotating \( R \) about the line \( x = 1 \)
Solution: This is similar but the meaning of \( r \) and \( R \) change.
Shell method gives: \( V = \int_0^1 2\pi(1-x)(\sqrt{x} - x^2) \, dx \)
Washer method gives \( V = \int_0^1 \pi((1-y^2)^2 - (1 - \sqrt{y})^2) \, dy \)
The other test had the same problem but with the line \( x = 2 \), in which case, 2 replaces 1 in the appropriate places.

(d) (10pt) The surface area of solid in part (b).
\[ SA = \int_0^1 2\pi r \, ds. \]
This applies to the top and bottom surface. These can be set up either as \( dx \) integrals or as \( dy \) integrals.
So setting this up as \( dx \) integral,
\[ SA_{top} = \int_0^1 2\pi x \sqrt{1 + ((\sqrt{x})')^2} \, dx = \int_0^1 2\pi x \sqrt{1 + 1/(4x)} \, dx. \]
And
\[ SA_{bottom} = \int_0^1 2\pi x \sqrt{1 + ((x^2)')^2} \, dx = \int_0^1 2\pi x \sqrt{1 + 4x^2} \, dx. \]
Then \( SA = SA_{top} + SA_{bottom} \)

If this is set up as a \( dy \) integral, for the top, use \( r = x = y^2 \) and \( ds = \sqrt{1 + (dx/dy)^2} \, dyf = \sqrt{1 + 4y^2} \, dy \) and for the bottom use \( r = x = \sqrt{y} \) and \( ds = \sqrt{1 + (dy/dx)^2} \, dx = \sqrt{1 + 1/4y} \, dx \)

Yet another way to do this: by symmetry, (if \( x \) and \( y \) are interchanged, the region remains the same), thus rotating this about the \( x \) axis gives the same solid as obtained by revolving about the \( y \) axis. If calculate the volume this way, for \( SA_{bottom} \), use \( r = y = \sqrt{x} \) and \( ds = \sqrt{1 + (dy/dx)^2} \, dx = \sqrt{1 + 1/4x} \, dx \) and for \( SA_{top} \) use \( r = y = x^2 \) and \( ds = \sqrt{1 + (dy/dx)^2} \, dy = \sqrt{1 + 4x^2} \, dx \).
2) (10pt) Let $T$ be the triangle with vertices $(2,0)$, $(5,0)$, $(5,h)$. Let $S$ be the solid obtained by rotating $T$ about the $y$-axis. Determine the volume of $S$.

This is easily done with theorem of Pappus: the cross sectional area is $3 \cdot h/2$. For the triangle, as we did in class, $\bar{x} = 4$ (it is $1/3$ of the height starting from a base of the triangle at $x = 5$). So $V = 2\pi \cdot 4 \cdot 3 \cdot h/2 = 12h\pi$. Of course this can be computed with integrals also, using the formula for the line $y = (h/3)(x - 2)$. In this case, $V = \int_2^5 2\pi x(h/3)(x - 2) \, dx$ works out to the same answer.

3) (10pt) A wire 30 cm long has a density that varies linearly from 1 gram/cm at the left end ($x = 0$) to 2 gram/cm at the right end ($x = 30$).

(a) what is its mass?

Solution: $m = \int_0^{30} \delta(x) \, dx$. So we need the density function $\delta$. This is a line that has the value 1 at 0 and 2 at 30. So the slope is 1/30 and $y$-intercept is 1. Thus $\delta(x) = x/30 + 1$. Now integrating, it is easy to get that $m = 45$ gm.

(b) where is the center of mass?

Solution: We need $\bar{x}$.

\[
\bar{x} = (\int_0^{30} x(x/30 + 1) \, dx) / m = ((x^3 / 30 + x^2 / 2)|_0^{30}) / 45 = 50/3 \text{ cm}
\]