In 1-4 circle the best answer. Each problem is worth 5 points.

1. The function \( f(x) = \frac{5 - x}{2 - x} \) has a horizontal asymptote at
   a) \( y = 1 \)  
   b) \( y = 5 \)  
   c) \( y = 2 \)  
   d) \( y = 5/2 \)  
   e) no horizontal asymptotes

2. How many points on the graph of \( f(x) = x^3 - 10x + 2 \) have a horizontal tangent line?
   a) 0  
   b) 1  
   c) 2  
   d) 3  
   e) infinitely many

3. If \( y = e^{x^2} \) then
   a) \( \frac{dy}{dx} = e^{2x} \)  
   b) \( \frac{dy}{dx} = x^2 e^{x^2 - 1} \)  
   c) \( \frac{dy}{dx} = ye^{-2x} \)  
   d) \( \frac{dy}{dx} = 2xe^{2x} \)  
   e) \( \frac{dy}{dx} = 2xe^{-x^2} \)

4. If \( f(x) = x \ln(x) \) then \( f'(x) = \)
   a) \( \frac{1}{x} \)  
   b) \( x \ln(1) + \ln(x) \)  
   c) \( 1 + \frac{1}{x} \)  
   d) \( 1 + \ln(x) \)  
   e) \( x \ln(1) \)

Questions 5-8 refer to the function \( f \) whose graph is given below. Simply answer the question, no explanation is required.

5. Give the intervals on which \( f' \) appears to be negative
   \((x_2, x_3), (x_3, 0)\)

6. List all points (x- coordinates) where \( f \) fails to be continuous
   \(x_3\)

7. List all points (x- coordinates) where \( f \) fails to have a derivative
   \(x_3, 0\)

8. List all intervals on which \( f'' \) appears to be positive
   \((x_4, x_5)\)

(See actual test, or graphs link for graph of function \( f \).)
9. (20pt) Let \( f(x) = 5x^{4/5} - 2x \).
   (a) Find all critical numbers. (These are values of \( x \) where \( f' \) is either zero or does not exist.)
   Solution: \( f' = 4x^{-1/5} - 2 \). There is a divide by zero at 0 so \( f' \) has a vertical tangent there. Thus \( x = 0 \) is one critical number. Setting \( f' = 0 \) gives
   \[
   4x^{-1/5} - 2 = 0
   \]
   \[
   x^{-1/5} = 1/2
   \]
   \[
   x = 32 \text{ (raise both sides of previous line to -5 power)}
   \]
   Thus 0, 32 are the critical numbers.

   (b) Find the absolute maximum and absolute minimum value of \( f \) on the interval \([-1, 32]\).
   Solution: We need to check the endpoints and the critical numbers: \( f(-1) = 5 + 2 = 7 \), \( f(0) = 0 \), \( f(32) = 80 - 64 = 16 \). Thus the absolute max of \( f \) is 16 and the absolute min is 0.

10. (10pt) Suppose \( x \) and \( y \) are differentiable functions of \( t \) and \( x^2 - y^2 = 5 \). Find \( \frac{dy}{dt} \) when \( x = 4 \), \( y = 3 \) and \( \frac{dx}{dt} = 2 \).
    Solution:
    \[
    2x \frac{dx}{dt} - 2y \frac{dy}{dt} = 0
    \]
    Now plug in known values:
    \[
    8 \cdot 2 - 6 \frac{dy}{dt} = 0
    \]
    \[
    \frac{dy}{dt} = 16/6 = 8/3
    \]
11. Let $f$ be a function with the following properties:

(i) $f(-2) = f(0) = 0$ and otherwise $f$ is nonzero

(ii) $f'(-1) = f'(1) = 0$ and otherwise $f'$ is nonzero

(iii) $f''$ is negative for $x < 0$, positive for $0 < x < 2$ and negative for $x > 2$.

(a) (10pt) Use the second derivative test to locate the $x$ coordinates of any local maxima and minima

Solution: Since $f'(1) = 0$, $f''(1) > 0$ $f$ has a local min at $x = 1$. Since $f'(-1) = 0$, $f''(-1) < 0$ $f$ has a local max at $x = -1$.

(b) (10pt) Sketch the graph of a function $f$ that satisfies conditions (i)-(iii).

Solution: Before trying to draw it, it helps to figure out where $f$ is increasing and decreasing. From part (a) $f$ has a local max at $x = -1$ so for the first derivative test to work, $f'$ needs to be positive for $x < -1$. Since $x = 1$ is local min, we need $f' < 0$ on $(-1, 1)$ and $f' > 0$ on $(1, \infty)$. It also helps to keep in mind that $f$ can not cross the $x$-axis if $x > 0$ by rule (i).

See graphs link for a sketch.

12. (10pt) A farmer has 80 feet of fence and wants to make a rectangular pen for some animals. A long barn can serve as one side of the pen and the fence serves as the other 3 sides. What dimensions of the pen (length of sides) maximize the area of the pen? (Hint: let $x$, $y$ be sides of the rectangle. What adds up to 80? What is to be maximized? Draw a picture.)

Solution: The fence will have 3 sides (the barn is the fourth side). So let $x$ be the length of the adjacent sides, and $y$ the length of the opposite side. The total length of fence is then $2x + y$ which must add up to 80 ft. Thus

$$2x + y = 80, \quad \text{or } y = 80 - 2x$$

The are inside the fence is $xy$ square ft. Using the above eq. for $y$,

$$A = xy = x(80 - 2x) = 80x - 2x^2$$

To maximize area, we take the derivative and set it to 0:

$$A' = 80 - 4x = 0$$

We obtain $x = 20$ and thus $y = 40$. 