CORRECTION to *"Central limit theorems for additive functionals of the simple exclusion process" Ann. Probab. (2000)* **28** *277-302 by S. Sethuraman*

Definition 2.1 in the above paper is incorrectly stated. In the proof of Theorem 2.1, which gives an invariance principle for certain processes satisfying Definition 2.1, conditions in Definition 2.1 are sufficient to deduce finite-dimensional convergence, but not enough to apply a maximal inequality for "demimartingales" to obtain tightness. The problem is Definition 2.1, as stated, only considers "pair increment associations" and not more general associations needed for the demimartingale property. We slightly strengthen the definition here in this note so that the proof of tightness in Theorem 2.1 holds. Details of how this is accomplished are given below.

By substituting the corrected Definition 2.1 for the previous one, all results in the article hold as written. In particular, Proposition 2.1, which is the link between Theorem 2.1 and the main results, and which states certain additive processes satisfy Definition 2.1, holds with the same argument.

Corrected Definition 2.1. Let $\{\overrightarrow{\mathbf{v}}(t) = (v_1(t), \ldots, v_m(t)) : t \geq 0\}$ *be an* m *dimensional* L^2 *process with stationary increments. We say* $\overrightarrow{\mathbf{v}}$ *has weakly positive associated increments if*

$$E[\phi(\overrightarrow{\mathbf{v}}(t+s) - \overrightarrow{\mathbf{v}}(s))\psi(\overrightarrow{\mathbf{v}}(s_1), \ldots \overrightarrow{\mathbf{v}}(s_n))] \ \geq \ E[\phi(\overrightarrow{\mathbf{v}}(t))]E[\psi(\overrightarrow{\mathbf{v}}(s_1), \ldots, \overrightarrow{\mathbf{v}}(s_n))]$$

for all coordinatewise increasing functions $\phi : \mathbb{R}^m \to \mathbb{R}$ *and* $\psi : (\mathbb{R}^m)^n \to \mathbb{R}$*, and all* $s, t \geq 0$*,* $0 \leq s_1 < \cdots < s_n = s$ *and* $n \geq 1$*.*

We remark the earlier Definition 2.1 only stipulated the pair condition

$$E[\phi(\overrightarrow{\mathbf{v}}(t+s) - \overrightarrow{\mathbf{v}}(s))\psi(\overrightarrow{\mathbf{v}}(s))] \ \geq \ E[\phi(\overrightarrow{\mathbf{v}}(t))]E[\psi(\overrightarrow{\mathbf{v}}(s))].$$

We now indicate how the modified definition applies in the proof of tightness in Theorem 2.1. Following standard tightness arguments, one needs to prove for a continuous mean-zero scalar process $v(t)$ with stationary increments satisfying corrected Definition 2.1, with $v(0) = 0$, $\lim_{t\to\infty} t^{-1}E[v(t)^2] = \sigma^2$ and $t^{-1/2}v(t) \Rightarrow \mathrm{N}(0, \sigma^2)$, that for all $\epsilon > 0$

$$\lim_{\delta \downarrow 0} \limsup_{\alpha\to\infty} \frac{1}{\delta} P\Big[\sup_{t\in[0,\delta]} |v(\alpha t)| > \epsilon\sqrt{\alpha}\Big] \ = \ 0. \tag{1}$$

For $\delta > 0$, let A be a countable dense set of $[0, \delta]$, and for $n \geq 1$, let A_n be a set of n points so that $A_n \uparrow A$. Fix also that $\delta \in A$ and $\delta \in A_1$. Then, for $\alpha \geq 1$, by continuity $\sup_{t\in[0,\delta]} |v(\alpha t)| = \sup_{t\in A} |v(\alpha t)|$, and for n large enough

$$P\Big[\sup_{t\in A} |v(\alpha t)| > \epsilon\sqrt{\alpha}\Big] \ \leq \ 2P\Big[\sup_{t\in A_n} |v(\alpha t)| > \epsilon\sqrt{\alpha}\Big].$$

Let now $0 \leq t_1 < \cdots < t_{n-1} < t_n = \delta$ be a labeling of A_n. From the corrected definition and mean-zero property $E[v(t)] = 0$ we have $E[(v(\alpha t_{j+1}) - v(\alpha t_j))\psi(v(\alpha t_j), \ldots, v(\alpha t_1))] \geq 0$

for all $1 \leq j \leq n-1$ and increasing ψ, and so $\{v(\alpha t) : t \in A_n\}$ is a demimartingale (cf. p. 362 [10]). Hence, we can apply the maximal inequality (Corollary 6 [10]) and variance convergence $\lim_{\alpha\to\infty}(\alpha\delta)^{-1}E[v(\alpha\delta)^2] = \sigma^2$ to get

$$\limsup_{\alpha\to\infty} P\Big[\sup_{t\in A_n}|v(\alpha t)| > \epsilon\sqrt{\alpha}\Big] \ \leq\ C_0\frac{\sigma\sqrt{\delta}}{\epsilon}\lim_{\alpha\to\infty}\Big\{P\Big[|v(\alpha\delta)| > \frac{\epsilon}{2}\sqrt{\alpha}\Big]\Big\}^{1/2}$$

for a universal constant C_0. From marginal convergence, $\lim_{\alpha\to\infty} P[|v(\alpha\delta)| > (\epsilon/2)\sqrt{\alpha}] = (2\pi\sigma^2)^{-1/2}\int_{(\epsilon/2)\delta^{-1/2}}^{\infty}\exp(-x^2/(2\sigma^2))dx$, and so (1) holds.

Also, we note typos in line 8, p. 281 change $2/(\pi\det(\sigma_p^2))$ to $1/(\pi(\det(\sigma_p^2))^{1/2})$; in lines 9-10, p. 286 ds to dr; in line 10, p. 293 $=$ to $\geq$; in line -1, p. 294 change > 0 to $< \infty$; in line -3, p. 297 $\exp(\lambda^2 s - \lambda - 1)(-\lambda s)$ should be $(\lambda^2 s - 2\lambda)\exp(-\lambda s)$.

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