The small world problem:
Six degrees of graph theory

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St. Olaf College Mathematics, Statistics and Computer Science
department seminar
Bacon game

Graphs

Origins

Erdős number

Many degrees

History

But, seriously...

Connections
Joint Work

This talk is based on joint work with

- Tom Bohman, Carnegie Mellon University
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- Tom Bohman, Carnegie Mellon University
- Alan Frieze, Carnegie Mellon University
Joint Work

This talk is based on joint work with

- Tom Bohman, Carnegie Mellon University
- Alan Frieze, Carnegie Mellon University
- Michael Krivelevich, Tel Aviv University
Six? degrees of separation

In the Kevin Bacon Game, it is postulated that the center of the Hollywood universe is

Kevin Bacon.
Six? degrees of separation

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This is false. It is Dennis Hopper.
Six? degrees of separation

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We link two actors together if they appeared together in the same movie.
Six degrees of separation

In the Kevin Bacon Game, it is postulated that the center of the Hollywood universe is

Kevin Bacon.

We link two actors together if they appeared together in the same movie.

(They must be together on a cast list at the IMDb.)
Bacon number

The actor’s Bacon number is the fewest number of steps it takes to connect that actor to
Bacon number

The actor’s # is the fewest number of steps it takes to connect that actor to Kevin Bacon.
The actor’s Bacon number is the fewest number of steps it takes to connect that actor to
Bacon number

An actor can have infinite Kevin Bacon number.
Bacon number

An actor can have infinite \( \# \).

(For example, an actor who appeared alone in only one film.)
Example: Kevin Costner

Kevin Costner is linked to Kevin Bacon because both appeared in JFK (1991).
Example: Kevin Costner

is linked to

because both appeared in
Example: Kevin Costner

Kevin Costner is linked to Kevin Bacon because both appeared in JFK (1991).

So, Kevin Costner’s Kevin Bacon number is ???
Example: Kevin Costner

Kevin Costner is linked to Kevin Bacon because both appeared in


So, Kevin Costner’s Kevin Bacon number is

1
Example: Kevin Costner

Kevin Costner is linked to Kevin Bacon because both appeared in *JFK* (1991).

So, Kevin Costner’s # is 1.
Illustration: Kevin Costner
Illustration: Kevin Costner
Illustration: Kevin Costner
Illustration: Kevin Costner
E.g.: Keanu “Woah!” Reeves

We know that

- appeared with in 
- appeared with in 
- appeared with in 
- appeared with in 
- appeared with in 

![Image of Keanu Reeves](image1)

![Image of Al Pacino](image2)

![Image of The Devil's Advocate poster](image3)
E.g.: Keanu “Woah!” Reeves

We know that

- Keanu Reeves appeared with Al Pacino in The Devil’s Advocate (1997)

- [Image of Keanu Reeves] appeared with [Image of Al Pacino] in
We know that

- Keanu Reeves appeared with Al Pacino in *The Devil’s Advocate* (1997)

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E.g.: Keanu “Woah!” Reeves
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We know that

- Keanu Reeves appeared with Al Pacino in *The Devil’s Advocate* (1997)
- Christopher Walken appeared with Courtney Love in *Basquiat* (1996)
- appeared with in
E.g.: Keanu “Woah!” Reeves

We know that

- Keanu Reeves appeared with Al Pacino in The Devil’s Advocate (1997)
- Al Pacino appeared with Christopher Walken in Gigli (2003)
- Christopher Walken appeared with Courtney Love in Basquiat (1996)
- Courtney Love appeared with Kevin Bacon in Trapped (2002)
A chain: Keanu "Neo" Reeves
A chain: Keanu “Neo” Reeves
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A chain: Keanu “Neo” Reeves
But it is also true that

- appeared with ??? inParenthood.
More: Keanu “Constantine” Reeves

But it is also true that

- appeared with [image] in [image].
More: Keanu “Constantine” Reeves

But it is also true that

- appeared with in *My Dog Skip*
More: Keanu “Constantine” Reeves

But it is also true that


Better: Keanu “Excellent!” Reeves
Better: Keanu “Excellent!” Reeves
Better: Keanu “Excellent!” Reeves
Better: Keanu “Excellent!” Reeves
Better: Keanu “Excellent!” Reeves
Better: Keanu “Excellent!” Reeves
Can we do even better?

has never appeared in a film with.
Can we do even better?

Kevin Bacon has never appeared in a film with Keanu Reeves.

So, is
Can we do even better?

Kevin Bacon has never appeared in a film with

So, Keanu Reeves’ Kevin Bacon number is

???
Can we do even better?

Kevin Bacon has never appeared in a film with

So, Keanu Reeves’ Kevin Bacon number is

2
In sum: Keanu “Bogus” Reeves
In sum: Keanu "Bogus" Reeves
In sum: Keanu “Bogus” Reeves
Experimental data

<table>
<thead>
<tr>
<th># of actors</th>
<th># of actors</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1806</td>
</tr>
<tr>
<td>2</td>
<td>145024</td>
</tr>
<tr>
<td>3</td>
<td>395126</td>
</tr>
<tr>
<td>4</td>
<td>95497</td>
</tr>
<tr>
<td>5</td>
<td>7451</td>
</tr>
</tbody>
</table>
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<tr>
<td>6</td>
<td>933</td>
</tr>
<tr>
<td>7</td>
<td>106</td>
</tr>
<tr>
<td>8</td>
<td>13</td>
</tr>
<tr>
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</tbody>
</table>
What about the high numbers?

As we said before, there are actors with infinite #.
What about the high numbers?

As we said before, there are actors with infinite degrees. The actors with large degrees are obscure and the reason why is fairly obvious.
Kevin Bacon not so special

Most successful actors follow the same pattern:
Kevin Bacon not so special

Most successful actors follow the same pattern:

For every pair of successful actors, they are connected by a path of length \( \leq 5 \)
Kevin Bacon not so special

Most successful actors follow the same pattern:

For every pair of successful actors, they are connected by a path of length $\leq 5$

Why?
Ask Christopher Walken

What do you think so far, Mr. Walken?
Ask Christopher Walken

What do you think so far, Mr. Walken?
Ask Christopher Walken

What do you think so far, Mr. Walken?

Pardon me?
Ask Christopher Walken

What do you think so far, Mr. Walken?
Our model

We will represent actors by VERTICES.
Our model

We will represent actors by VERTICES
Our model

We will represent actors by VERTICES

and connect them with EDGES
Our model

We will represent actors by VERTICES

and connect them with EDGES

if they appeared in the same film.
Our model

We will represent actors by VERTICES

\[ \bullet \]

and connect them with EDGES

\[ \bullet \quad \text{——} \quad \bullet \]

if they appeared in the same film.

This is a GRAPH.
Model parameters

- There are $n$ actors.
Model parameters

- There are $n$ actors.
- Fix a constant $d$. 
Model parameters

- There are \( n \) actors.
- Fix a constant \( d \).
- We will begin with an \textit{arbitrary} graph \( H \) such that . . .
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$$0.1$$
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  $0.1, \quad 0.01$
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- . . . in $H$, each actor is connected to at least $dn$ other actors.
- The constant $d$ can be extremely tiny:
  
  $$0.1, \quad 0.01, \quad 10^{-10^{100}}$$

It just needs to be independent of $n$. 
Random casting

We add $f(n)$ random casting connections.
Random casting

We add $f(n)$ random casting connections.

What does RANDOM mean?
Random edges

Let $N$ be the number of pairs with no connection between them (NON-EDGES). We can create $m$ new random edges in the following way:
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The question:
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The question:

What is the longest distance between any pair of actors?
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**The question:**

*What is the longest distance between any pair of actors (DIAMETER)*?
Proposition

**Proposition.** If \( f(n) \to \infty \) as \( n \to \infty \), then
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$$\Pr(diam \leq \ ) \to 1$$
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$$\Pr(\text{diam} \leq 7) \to 1$$
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\Pr(\text{diam} \leq 7) \to 1
\]

**Important points:**

• Recall \( f(n) \) is the number of random connections.
• “7” doesn’t depend on \( d \) at all.
• \( f(n) \) can be very small: \( \sqrt{n}, \log n, \sqrt{\log \log \log n} \).
**Proposition**

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Proof of diameter 7

This is not too hard to prove.
Proof of diameter 7

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Take $G$ and construct $v_1$. 
Proof of diameter 7

This is not too hard to prove.
Take $G$ and construct $v_1$, $v_2$. 
Proof of diameter 7

This is not too hard to prove.
Take $G$ and construct $v_1, v_2, v_3$, 

\[ \text{Take } G \text{ and construct } v_1, v_2, v_3, \]
Proof of diameter 7

This is not too hard to prove.
Take $G$ and construct $v_1, v_2, v_3, \ldots$
Proof of diameter 7

Take $G$ and construct $v_1, v_2, v_3, \ldots$ such that $\text{dist}(v_i, v_j) \geq 3$ for $i \neq j$. 
Proof of diameter 7

So, since they are of distance 3,
Proof of diameter 7

So, since they are of distance 3, their neighborhoods do not intersect.
Proof of diameter 7

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The process will end after $\leq \lfloor n/(dn + 1) \rfloor$ steps.
Proof of diameter 7

The process will end after \( \leq \lfloor n/(dn + 1) \rfloor \) steps.

The random edges guarantee at least one edge between each pair of the neighborhoods.
The path of length 7

So, every vertex is of distance at most 2 from some $v_i$. 
The path of length 7

So, every vertex is of distance at most 2 from some $v_i$. 

\begin{center}
\begin{tikzpicture}
  \node [fill=red] (v1) at (0,0) {}; 
  \node [fill=blue] (u) at (-2,-2) {u}; 
  \node [fill=blue] (w) at (2,-2) {w}; 
  \node [fill=red] (v) at (0,-4) {$v_i$}; 
  \node [fill=red] (v') at (0,-8) {$v_j$}; 
  \draw (v1) -- (u); 
  \draw (v1) -- (v); 
  \draw (v1) -- (v'); 
  \draw (v) -- (v'); 
  \end{tikzpicture}
\end{center}
So, every vertex is of distance at most 2 from some $v_i$. One edge is between each pair of neighborhoods.
The path of length 7

So, every vertex is of distance at most 2 from some $v_i$. One edge is between each pair of neighborhoods.
The path of length 7

So, every vertex is of distance at most 2 from some \( v_i \). One edge is between each pair of neighborhoods. And this gives the path of length 7:

![Graph diagram](image)
The path of length 7

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Even better

There’s a better result, which is harder to prove:
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**Theorem.** [BFKM] If $f(n) \to \infty$ as $n \to \infty$, then

$$\Pr(\text{diam} \leq 5) \to 1$$
**Even better**

There’s a better result, which is harder to prove:

**Theorem.** [BFKM] If $f(n) \to \infty$ as $n \to \infty$, then

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To prove the theorem, you need the **Regularity lemma**.
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The Regularity lemma is Endre Szemerédi’s powerful and complicated graph theoretic tool.
Best possible?

The theorem is “tight”:
Best possible?

The theorem is “tight”:

*If there aren’t an infinite number of edges added, then some $H$’s will be disconnected.*
What about closer connections?

- To get $\text{diam} \leq 4$, you need $c_1 \log n$ random connections.
- To get $\text{diam} \leq 3$, you need $c_1 \log n$ random connections.
- To get $\text{diam} \leq 2$, you need $c_2 \log n$ random connections.
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• To get $\text{diam} \leq 3$, you need $c_1 \log n$ random connections.

• To get $\text{diam} \leq 2$, you need $c_2 n \log n$ random connections.
5 degrees!

So, if you have a system, with a linear minimum degree and a little bit of randomness, the diameter will go to 5!
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So, if you have a system, with a linear minimum degree and a little bit of randomness, the diameter will go to 5!

What do you think of that?
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What do you think of that?

I see.
5 degrees!

So, if you have a system, with a linear minimum degree and a little bit of randomness, the diameter will go to 5!

What do you think of that?

Let’s move to a new problem.
Origins of the problem

The original question posed by Stanley Milgram
Origins of the problem

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The original question posed by Stanley Milgram asked what the average distance was among people in a network.
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But his assertion that the average distance was around 6 stuck.
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But his assertion that the average distance was around 6 stuck.
Advantages and disadvantages

Our model has some advantages and some disadvantages.

Disadvantages:
Advantages and disadvantages

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**Disadvantages:**

- Each person must be connected to at least $dn$ other people.
Advantages and disadvantages

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• Not good for modeling the Internet.
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**Advantages:**

- Very weak restriction on structure.
- The number of random connections is tiny.
- An upper bound of 5 on the diameter.
  (Since diameter is maximum distance, it is always at most the average distance.)
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Another model

Another model due to Fan Chung and Linyuan Lu
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The Chung-Lu model

Advantages:
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Disadvantages:

• The number of random connections is huge.

• Weak result: The average distance is \( \approx \frac{\log_2 n}{\log_2 \tilde{d}} \), where \( \tilde{d} \) relates to the average degree (number of connections).
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Erdős number

One of the most prolific mathematicians of the 20th century was Paul (Pál) Erdős

March 26, 1913-September 20, 1996
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Paul (Pál) Erdős
March 26, 1913-September 20, 1996
Erdős number project

The Erdős number project is concerned with the distance of mathematicians from Paul Erdős.
Erdős number project

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Erdős number project

The project is concerned with the distance of mathematicians from Paul Erdős.

Two mathematicians are connected if they co-authored a paper together and that paper appears in Mathematical Reviews, accessible by MathSciNet.
**Most prolific authors**

- **Paul Erdős**: 1401 papers (Erdős number 0)
- **Drumi Bainov**: 782 (Erdős number 4)
- **Leonard Carlitz**: 730 (Erdős number 2)
- **Lucien Godeaux**: 644 (Erdős number ∞)
- **Saharon Shelah**: 600 (Erdős number 1)
Most prolific authors

- : 1401 papers (Erdős number 0)
- Drumi Bainov: 782 (Erdős number 4)
- Leonard Carlitz: 730 (Erdős number 2)
- Lucien Godeaux: 644 (Erdős number ∞)
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Erdős number statistics

- wrote 1401 papers in Math Reviews.
Erdős number statistics

- wrote 1401 papers in Math Reviews.
- There are 337,000 vertices (authors) in the graph.
Erdős number statistics

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- There are 337,000 vertices (authors) in the graph.
- There are about 496,000 edges.
Erdős number statistics

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- There are 337,000 vertices (authors) in the graph.
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- Average number of authors per paper: 1.45
Erdős number statistics

- wrote 1401 papers in Math Reviews.
- There are 337,000 vertices (authors) in the graph.
- There are about 496,000 edges.
- Average number of authors per paper: 1.45
- Average number of papers per author: 6.87
## Experimental data

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(Most recent data)
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(R. G. Kamalov)
The unknown mathematician
The unknown mathematician
The unknown mathematician
The unknown mathematician
The unknown mathematician
The unknown mathematician
Applying our model

To have it be very likely that everyone is connected by a path of no more than 5 acquaintances, just arrange a few random meetings.
Applying our model

To have it be very likely that everyone is connected by a path of no more than 5 acquaintances, just arrange a few random meetings.

Think about people at St. Olaf.
St. Olaf cliques

Mathematicians
St. Olaf cliques
More St. Olaf cliques

But for us to get diameter $\leq 5$, we do need each person to know at least $dn$ others before we add few random edges.
But for us to get diameter \( \leq 5 \), we do need each person to know at least \( dn \) others before we add few random edges.
Back (to) Bacon

Let us return to the Kevin Bacon question.
Back (to) Bacon

Let us return to the Kevin Bacon question.

We want to find actors with an

- INFINITE
- with \# = 8.
Infinite Kevin Bacon number

is someone with infinite #.

Thomas Alva Edison only appeared in ONE MOVIE (a brief documentary) and was the only actor.
Infinite Kevin Bacon number

is someone with infinite #.

Thomas Alva Edison only appeared in ONE MOVIE (a brief documentary) and was the only actor.

Not soon coming to DVD:
Infinite Kevin Bacon number

is someone with infinite #.

Thomas Alva Edison only appeared in ONE MOVIE (a brief documentary) and was the only actor.

Not soon coming to DVD: MR. EDISON AT WORK IN HIS CHEMICAL LABORATORY (1897).
Kevin Bacon number 7

Joseph Wheeler appeared in two films:

- **General Wheeler and Secretary of War Alger at Camp Wikoff** (1898), a short documentary, in which he appeared with Russell Alexander Alger (Kevin Bacon number 6)
Kevin Bacon number 7

Joseph Wheeler appeared in two films:

- **General Wheeler and Secretary of War Alger at Camp Wikoff** (1898), a short documentary, in which he appeared with Russell Alexander Alger (Kevin Bacon number 6)

- **Surrender of General Toral** (1898), again, a short documentary, with William Rufus Shafter.
Kevin Bacon number 8

William Rufus Shafter also appeared in two films:

- *Surrender of General Toral* (1898) with Joseph Wheeler.
William Rufus Shafter also appeared in two films:

- **Surrender of General Toral** (1898) with Joseph Wheeler.

- **Major General Shafter** (1898) as the only credited cast member.
Conclusion

- Russell Alexander Alger has $\# = 6$, $\# = 7$, and $\# = 8$. 
Conclusion

- Russell Alexander Alger has $# = 6$,
- Joseph Wheeler has $# = 7$, and
Conclusion

• Russell Alexander Alger has $\# = 6$.

• Joseph Wheeler has $\# = 7$, and

• William Rufus Shafter has $\# = 8$. 
8. William Rufus Shafter was in *Surrender of General Toral* (1898) with Joseph Wheeler
The chain

8 William Rufus Shafter was in *Surrender of General Toral* (1898) with Joseph Wheeler

7 Joseph Wheeler was in *General Wheeler and Secretary of War Alger at Camp Wikoff* (1898) with Russell Alexander Alger
The chain

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7 Joseph Wheeler was in **General Wheeler and Secretary of War Alger at Camp Wikoff** (1898) with Russell Alexander Alger

6 Russell Alexander Alger was in **President McKinley’s Inspection of Camp Wikoff** (1898) with President William McKinley
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2 Walter McGrail was in **Dick Tracy vs. Crime Inc.** (1941) with Wally Rose

1 Wally Rose was in **Murder in the First** (1995) with
The chain

2 Walter McGrail was in **DICK TRACY vs. CRIME INC.** (1941) with Wally Rose

1 Wally Rose was in **MURDER in the FIRST** (1995) with
Presidential precedents

William McKinley has a non-cinematic distinction.
Presidential precedents

William McKinley has a non-cinematic distinction.

He was one of four presidents to be assassinated:
William McKinley has a non-cinematic distinction.

He was one of four presidents to be assassinated:

McKinley
Sep. 14, 1901
Presidential precedents

William McKinley has a non-cinematic distinction.

He was one of four presidents to be assassinated:

McKinley
Sep. 14, 1901

Kennedy
Nov. 22, 1963
President precedents

William McKinley has a non-cinematic distinction.

He was one of four presidents to be assassinated:

- Lincoln
  Apr. 15, 1865
- McKinley
  Sep. 14, 1901
- Kennedy
  Nov. 22, 1963
Presidential precedents

William McKinley has a non-cinematic distinction.

He was one of four presidents to be assassinated:

Lincoln
Apr. 15, 1865

Garfield
Sep. 19, 1881

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Lincoln
Apr. 15, 1865

Garfield
Sep. 19, 1881

McKinley
Sep. 14, 1901

Kennedy
Nov. 22, 1963

Dude, what a downer. Let’s at least focus on the wacky one.
Garfield (not the cat)
Garfield (not the cat)

- Born in a log cabin in 1931.
Garfield (not the cat)

- Born in a log cabin in 1931 near Cleveland.
Garfield (not the cat)

- Born in a log cabin in 1931 near Cleveland.
- 18 years in the House.
Garfield (not the cat)

- Born in a log cabin in 1931 near Cleveland.
- 18 years in the House.
- Elected in 1880.
Garfield (not the cat)

- Born in a log cabin in 1931 near Cleveland.
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- Shot on July 2.

• Amateur mathematician.
Garfield (not the cat)

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- Shot on July 2, died on September 19.
- Amateur mathematician.
Published mathematician

As a Congressman, Garfield got a publication credit:
Published mathematician

As a Congressman, Garfield got a publication credit:

Published mathematician

As a Congressman, Garfield got a publication credit:


Garfield found a proof of the Pythagorean theorem:
As a Congressman, Garfield got a publication credit:


Garfield found a proof of the Pythagorean theorem:
Garfield's proof

area of trapezoid = area of triangle 1 + area of triangle 2 + area of triangle 3
Garfield’s proof

area of trapezoid  =  area of triangle 1

+ area of triangle 2

+ area of triangle 3
Garfield’s proof

area of trapezoid = area of triangle 1

+ area of triangle 2

+ area of triangle 3
Garfield’s proof

\[ \frac{1}{2}(a + b)(a + b) = \text{area of triangle 1} \]

\[ + \text{area of triangle 2} \]

\[ + \text{area of triangle 3} \]
Garfield’s proof

\[ \frac{1}{2}(a + b)(a + b) = \text{area of triangle 1} \]

\[ + \text{ area of triangle 2} \]

\[ + \text{ area of triangle 3} \]
Garfield’s proof

\[
\frac{1}{2} (a + b)(a + b) = \frac{1}{2} c^2
\]

\[
+ \text{ area of triangle 2}
\]

\[
+ \text{ area of triangle 3}
\]
Garfield's proof

\[
\frac{1}{2} (a + b)(a + b) = \frac{1}{2} c^2 \\
+ \text{area of triangle 2} \\
+ \text{area of triangle 3}
\]
Garfield’s proof

\[ \frac{1}{2}(a+b)(a+b) = \frac{1}{2}c^2 \]

\[ + \frac{1}{2}ab + \text{area of triangle 3} \]
Garfield’s proof

\[
\frac{1}{2}(a + b)(a + b) = \frac{1}{2}c^2
\]

\[
+ \ \frac{1}{2}ab
\]

+ area of triangle 3
Garfield’s proof

\[
\frac{1}{2} (a + b)(a + b) = \frac{1}{2} c^2 \\
+ \frac{1}{2} ab \\
+ \frac{1}{2} ab
\]
Garfield’s proof

\[
\frac{1}{2} (a + b)(a + b) = \frac{1}{2} c^2 \\
+ \frac{1}{2} ab \\
+ \frac{1}{2} ab
\]
Garfield’s proof

\[
\frac{1}{2}(a + b)(a + b) = \frac{1}{2}c^2 + \frac{1}{2}ab + \frac{1}{2}ab
\]
Garfield’s proof

\[(a + b)(a + b) = c^2 + ab + ab\]
Garfield’s proof

\[ a^2 + 2ab + b^2 = c^2 + ab + ab \]
Garfield’s proof

\[ a^2 + b^2 = c^2 \]
Computer networks

Graphs model much more serious stuff.

I.e.,

- computer networks,
Computer networks

Graphs model much more serious stuff.

I.e.,

- computer networks,

- shipping routes,
Computer networks

Graphs model much more serious stuff.

I.e.,

- computer networks,
- shipping routes,
- distribution networks.
Network question

In networks we are concerned with one particular quantity:
Network question

In networks we are concerned with one particular quantity:

**CONNECTIVITY:** A connected graph is \( k \)-connected if removing any set of \( k - 1 \) vertices (and all relevant edges) leaves the graph connected.
Same model

- $n$ computers
Same model

• $n$ computers

• in $H$, each computer is connected to $\geq dn$ others
Same model

- $n$ computers
- In $H$, each computer is connected to $\geq dn$ others
- Add $f(n)$ random connections
Same model

- \( n \) computers

- in \( H \), each computer is connected to \( \geq \ d n \) others

- add \( f(n) \) random connections

Of course, we want high connectivity with as little randomness as possible.
Connectivity theorem

Theorem. [BFKM] Let $k$ be a function of $n$ that is $\ll n$. 
Connectivity theorem

**Theorem.** [BFKM] Let $k$ be a function of $n$ that is $\ll n$. (That is, $k$ grows more slowly than $n$ as $n \to \infty$.)
Connectivity theorem

**Theorem.** [BFKM] Let $k$ be a function of $n$ that is $\ll n$. (That is, $k$ grows more slowly than $n$ as $n \to \infty$.) Let $H$ have the property that each vertex is connected to at least $dn$ other vertices.
Connectivity theorem

**Theorem.** [BFKM] Let $k$ be a function of $n$ that is $\ll n$. (That is, $k$ grows more slowly than $n$ as $n \to \infty$.) Let $H$ have the property that each vertex is connected to at least $dn$ other vertices.

- If $f(n) \gg k$, then the graph becomes $k$-connected, with high probability.
Connectivity theorem

**Theorem.** [BFKM] Let $k$ be a function of $n$ that is $\ll n$. (That is, $k$ grows more slowly than $n$ as $n \to \infty$.) Let $H$ have the property that each vertex is connected to at least $dn$ other vertices.

- If $f(n) \gg k$, then the graph becomes $k$-connected, with high probability.

- If $d < 1/2$, there is an $H_0$ such that for every $k \ll n$, $f(n) = k - 1$ ensures that the graph fails to be $k$-connected, with high probability.
Bottom line

A way to interpret this theorem is:
Bottom line

A way to interpret this theorem is:

*If you need $k$-connectivity,*
A way to interpret this theorem is:

\textit{If you need }$k$\textit{-connectivity,}

\textit{then you need to add a little more (asymptotically) random edges than }$k$. 
Bottom line

A way to interpret this theorem is:

*If you need $k$-connectivity,*

*then you need to add a little more (asymptotically) random edges than $k$.*

If fewer than $k$ random edges are added, $k$-connectivity does not necessarily occur.
Worst case

What is that $H_0$?

$H_0 =$

Disjoint cliques give the worst case.
Other properties

We’ve used this model to investigate other properties:

- Hamilton cycle
We’ve used this model to investigate other properties:

- Hamilton cycle
- Small cliques as subgraphs
Other properties

We’ve used this model to investigate other properties:

- Hamilton cycle
- Small cliques as subgraphs
- Chromatic number
Other properties

We’ve used this model to investigate other properties:

- Hamilton cycle
- Small cliques as subgraphs
- Chromatic number

Some use the Regularity lemma, some use other techniques.
And so much more...

This is just a small taste of what graph theory can do.
And so much more...

This is just a small taste of what graph theory can do.
And so much more...

This is just a small taste of what graph theory can do.

The Internet
And so much more...

This is just a small taste of what graph theory can do.

The Internet
(not to scale)
And so much more...

Thank you very much!

The Internet

(not to scale)
More Cowbell!
Bacon and Erdős
Bacon and Erdős
Bacon and Erdős
Bacon and Erdős

How are these guys related?
Celebrity nerds

There are some mathematicians and physicists who have Bacon numbers and low Erdős numbers:

- Brian Greene (Bacon number = 3, Erdős number = 2) in *Frequency* (2000).
- Dave Bayer (Bacon number = 3, Erdős number = 2) in *A Beautiful Mind* (2001).
Celebrity nerds

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There are some mathematicians and physicists who have Bacon numbers and low Erdős numbers:

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Nerd celebrities

Danica McKellar

Nerd celebrities

Danica McKellar, math nerd.
Nerd celebrities

Danica McKellar, math nerd. Best known for: The Wonder Years (1988-1993) and
Nerd celebrities

Nerd celebrities

Danica McKellar was in *Intermission* (2004) with Susan Leslie.

Susan Leslie was in *Beauty Shop* (2005) with [Image]
Nerd celebrities

- Danica McKellar was in Intermission (2004) with Susan Leslie.
Nerd celebrities

- Danica McKellar was in Intermission (2004) with Susan Leslie.

- Susan Leslie was in Beauty Shop (2005) with [Image of Kevin Bacon]
Danica’s Math Career

4 Danica McKellar wrote

Danica’s Math Career

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3 Lincoln Chayes wrote

“No directed fractal percolation in zero area”, which appeared in The Journal of Statistical Physics, with Peres and Pemantle.
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Erdős, Hollywood Leading Man

Paul Erdős was in \textit{N Is a Number} (1993) with Gene Patterson
Erdős, Hollywood Leading Man

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Erdős, Hollywood Leading Man

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