M608X (S08) Homework 1

Due date: 7 February 2008

Do at least 8 of the following:

1. Prove Hall's theorem from König's theorem.

2. Prove that Egerváry's statement is equivalent to that of König.

3. Prove that $T_{n,r}$ is the n-vertex r-partite graph with the most number of edges.

4. For each n, find two examples of graphs with minimum degree at least $\lceil n/2 \rceil - 1$ which are not Hamiltonian.

5. Prove that if G is a simple graph on n vertices with minimum-degree at least $n/2$, then G contains a matching (1-regular subgraph) with $\lfloor n/2 \rfloor$ edges.

6. Prove that if G has a nearly equitable $(r+1)$-coloring f in which the large class V^+ is accessible, then G has an equitable $(r+1)$-coloring.

7. Prove the complementary form of Hajnal-Szemerédi, given the original form.

8. Prove the general form of Ramsey's theorem.

9. Let G be a 3-regular graph. Prove that there exists a bipartition of $V(G) = A + B$ such that at least two-thirds of the edges of G have on endvertex in A and one endvertex in B.

10. Let G be a graph on n vertices with degree sequence $d_1 \leq d_2 \leq \cdots \leq d_n$. Prove that there is a partition of $V(G) = A_1, \ldots, A_k$ such that at least

$$\frac{1}{2}\sum_{i=1}^{n}\left\lceil \frac{k-1}{k} d_i \right\rceil = e(G) - \frac{1}{2}\sum_{i=1}^{n}\left\lfloor \frac{d_i}{k} \right\rfloor$$

    edges have endpoints in distinct parts.

11. Write a "good" homework problem based on the first three weeks of material.