Do at least 2 of the following:

(1) Use the symmetric version of the Lovász Local Lemma to prove that \( R(k, k) > \sqrt{2} (1 - o(1)) k^{2k/2} \). This is an improvement of a multiplicative factor of 2 over the naive probabilistic bound in Erdős’ 1947 paper.

(2) An arithmetic progression of length \( k \) is a sequence \( a, a+d, a+2d, \ldots, a+(k-1)d \). The van der Waerden number \( W(k) \) is the least \( n \) so that if \( \{1, \ldots, n\} \) is two-colored, then it has a monochromatic arithmetic progression of length \( k \).

Use the Lovász Local Lemma to prove that \( W(k) > (1 - o(1)) \frac{2^{k-1}}{ek} \).

(3) Prove the Slicing Lemma for \( \epsilon \)-regular pairs:

**Lemma (Slicing Lemma)** Let \( (A, B) \) be an \( \epsilon \)-regular pair with density \( d \). If \( A' \subseteq A \) and \( B' \subseteq B \) such that \(|A'| \geq \epsilon |A|\) and \(|B'| \geq \epsilon |B|\), then \( (A', B') \) is an \( \epsilon' \)-regular pair, with \( \epsilon' = \max \left\{ 2\epsilon, \frac{|A'|}{|A|}, \frac{|B'|}{|B|} \right\} \) and density in \([d - \epsilon, d + \epsilon]\).