M606 - Posets - LYM

Note Title 3/29/2007

<u>Definition</u>

The inequality $\sum_{x \in A} N_{r(x)}^{-1} \le 1$ is the <u>LYM inequality</u>. A graded poset satisfies the <u>LYM property</u> if each antichain A satisfies LYM. A poset that satisfies LYM is an <u>LYM order</u>.

[Lubell, 1966] [Yamamoto, 1954] [Meshalkin, 1963]

<u>Theorem</u> All LYM orders are Sperner.

<u>Proof</u> Let r^* have the largest rank.

Let A be an antichain.

$$1 \ge \sum_{x \in A} N_{r(x)}^{-1} \ge \sum_{x \in A} N_{r^*}^{-1} = |A| \, N_{r^*}^{-1}$$

Which implies $N_{r^*} \geq |A|$. $\square$

In fact,

<u>Thm</u> [Erdős, 1945]

LYM $\Rightarrow$ Strong Sperner

The proof takes us too far afield.

[Recall that SCO $\Rightarrow$ strong Sperner.]

Erdős originally proved Boolean lattice is strong Sperner, but the proof works for any LYM order.

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