Homework 5

Spring 2009 M606:
Enumerative Combinatorics and Partially Ordered Sets

Due Apr. 21, 2009, assigned Apr. 14, 2009

You may ask me any questions in office hours.
LaTeX or other typed solutions are very strongly preferred.

1 Do all three (3) of the following:

PROBLEM 1 *Let $D(N)$ be the poset of divisors of the positive integer $N = p_1^{a_1} p_2^{a_2} \cdots p_r^{a_r}$.*

a. *Prove that $D(N)$ is a product of chains. What is the length of these chains?*

b. *Use this to show that $D(N)$ is a symmetric chain order.*

c. *If $N = p_1^{a_1} p_2^{a_2}$, what is the largest Whitney number?*

PROBLEM 2 *Let L be a lattice and x, y, z be elements in that lattice. Prove the following:*

a. $x \wedge y \leq x \leq x \vee z$.

b. *If $x \leq z$, then $x \wedge y \leq z \wedge y$ and $x \vee y \leq z \vee y$.*

c. *(4-point Lemma) If $z, w \leq x, y$, then $z \vee w \leq x \wedge y$.*

d. $(x \wedge y) \vee (x \wedge z) \leq x \wedge (y \vee z)$.

e. *If $z \leq x$, then $x \wedge (y \vee z) \geq (x \wedge y) \vee z$.*

PROBLEM 3 *Recall that a partition of an integer n is a sequence $a_1 \geq \cdots \geq a_k$ such that $n = a_1 + \cdots + a_k$. Prove that the poset of partitions of integer n ordered by dominance is a lattice. In this poset $\mu \leq \lambda$ if $\sum_{i=1}^{j} \mu_i \leq \sum_{i=1}^{j} \lambda_i$ for all j.* ***Hint:*** *You only need to show the existence of $\hat{1}$ and a well-defined meet operation.*