Take-home Exam 1

Spring 2009 M606:
Enumerative Combinatorics and Partially Ordered Sets

Due Mar. 5, 2009, assigned Mar. 3, 2009

Please hand in solutions for all problems. You must work entirely on your own, but you may use class notes. You may also use the notes I post on the web or the textbooks mentioned in the syllabus. Use no other sources.

LaTeX or other typed solutions are preferred, but neat writing is sufficient.

PROBLEM 1 (10 points) *Prove that, for all nonnegative integers x and n,*

$$\sum_{i=0}^{n} \binom{x+i}{i} = \binom{x+n+1}{n}.$$

PROBLEM 2 (10 points) *How many paths are there in the plane from $(0,0)$ to $(m,n) \in \mathbb{N} \times \mathbb{N}$, if each step in the path is of the form $(1,0)$ or $(0,1)$? In other words, each step is due east or due north. Give a combinatorial proof.*

PROBLEM 3 (10 points) *Solve the following difference equation:*

$$\begin{cases} a_n = 2a_{n-1} + 4a_{n-2} - 8a_{n-3}, & \text{for } n \geq 3; \\ a_0 = 2, a_1 = 2, a_2 = 0 \end{cases}$$

Note: *you must show your derivation according to methods developed in class. It is not sufficient to show that a given solution satisfies the recurrence.*

PROBLEM 4 (10 points) *Let $a_n = \frac{1}{n}$ for $n \geq 1$ and $a_0 = 0$. Compute the ordinary generating function $\sum_{n\geq 0} a_n x^n$.*

PROBLEM 5 (10 points) *Let $f \overset{\text{ops}}{\leftrightarrow} \{a_n\}_0^\infty$. Give the sequence whose ordinary power series is $f/(1-x)$.*