The following answers are for the sample Calculus Placement Exam.

ANSWERS (Calculus)

1. $f(x) = 1 - 3\cos x$
   $f'(x) = -3(-\sin x) = 3\sin x$
   $f'\left(\frac{\pi}{2}\right) = 3 \cdot 1 = 3$

2. $f(x) = \sin(\pi x)$
   $f'(x) = \pi \cos(\pi x)$
   $f''(x) = -\pi^2 \sin(\pi x)$
   $f'\left(\frac{1}{2}\right) = -\pi^2 \sin\frac{\pi}{2} = -\pi^2$

3. a) $f(x) = \sqrt{1 + \sin x}$
   $f'(x) = \frac{\cos x}{2\sqrt{1+\sin x}}$

   b) $f(x) = \tan\left(\frac{\pi}{x^2}\right)$
   $f'(x) = \sec^2\left(\frac{\pi}{x^2}\right) \cdot \frac{d}{dx}\left(\frac{\pi}{x^2}\right)$
   $= \frac{-2\pi}{x^3}\sec^2\left(\frac{\pi}{x^2}\right)$

   c) $f(x) = \frac{\sin x}{x^2 + 2}$
   $f'(x) = \frac{(x^2+2)\cos x - 2x\sin x}{(x^2+2)^2}$

   d) $f(x) = x\cos 2x$
   $f'(x) = x \cdot \frac{d}{dx}\cos 2x + (\cos 2x) \cdot \frac{dx}{dx}$
   $= -2x\sin 2x + \cos 2x$

4. $f(x) = \sqrt{x+1}$
   $f'(x) = \frac{1}{2\sqrt{x+1}}$
   $f'(3) = \frac{1}{2\sqrt{3+1}} = \frac{1}{4}$
   $\therefore$ tangent line is $y - 2 = \frac{1}{4}(x-3)$

5. a) $\lim_{x\to 3}\frac{x^2 - 6x + 9}{x^2 - 2x - 3}$
   $= \lim_{x\to 3}\frac{(x-3)^2}{(x-3)(x+1)}$
   $= \lim_{x\to 3}\frac{x-3}{x+1}$
   $= 0$

   b) $\lim_{x\to 1^-}\frac{|x-1|}{x-1}$
   $= \lim_{x\to 1^-}\frac{1-x}{x-1}$
   $= \lim_{x\to 1^-}\frac{-(x-1)}{x-1}$
   $= -1$

   c) $\lim_{x\to 0}\frac{x}{\sin(\pi x)}$
   $= \lim_{x\to 0}\frac{\pi x}{\sin(\pi x)} \cdot \frac{1}{\pi}$
   $= 1 \cdot \frac{1}{\pi} = \frac{1}{\pi}$

6. a) $h(x) = \frac{x}{x^2+1}$
   $h'(x) = \frac{(x^2+1)(1) - x(2x)}{(x^2+1)^2}$
   $= \frac{-x^2+1}{(x^2+1)^2}$
   $h'(x) > 0$ when $-x^2 + 1 > 0$ or $x^2 < 1$; therefore $-1 < x < 1$

   b) $h'(x) = 0$ when $-x^2 + 1 = 0$ or $x = \pm 1$. At $x = +1$ there is a relative maximum because $h'(x) > 0$ to the left of 1 and $h'(x) < 0$ to the right of 1. At $x = -1$, there is a relative minimum because $h'(x) < 0$ to the left of -1, but $h'(x) > 0$ right of -1.

c) $\lim_{x\to\infty} h(x)$

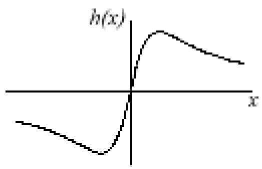
$$= \lim_{x\to\infty} \frac{x}{x^2+1}$$

$$= \lim_{x\to\infty} \frac{1}{x+\frac{1}{x}}$$

$$= 0$$

Hence, $y=0$ is a horizontal asymptote.

d)



7. $\lim_{x\to\frac{\pi}{2}} f(x) = \lim_{x\to\frac{\pi}{2}} \frac{1}{\sec x} = \lim_{x\to\frac{\pi}{2}} \cos x = 0$

Use $f\left(\frac{\pi}{2}\right) = 0$.

8. $-3\sin x = \tan y + y$

Differentiate implicitly:

$$-3\cos x = \left(\sec^2 y\right)\frac{dy}{dx} + \frac{dy}{dx}$$

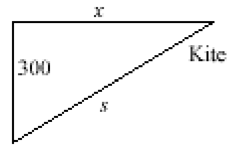
When $x=0$, $y=0$, so

$$-3\cos 0 = \left(\sec^2 0\right)\frac{dy}{dx} + \frac{dy}{dx}$$

$$-3 = 2\frac{dy}{dx}$$

$$\left.\frac{dy}{dx}\right|_{(0,0)} = -\frac{3}{2}$$

9. $s^2 = 300^2 + x^2$.



Differentiate with respect to time *t*:

$2s\frac{ds}{dt} = 2x\frac{dx}{dt}$. We want $\frac{ds}{dt}$ when $s=500$ and $\frac{dx}{dt}=10$. So, $\frac{ds}{dt} = \frac{x}{s}\frac{dx}{dt} = \frac{\sqrt{s^2-300^2}}{s}\frac{dx}{dt}$

$$= \frac{\sqrt{500^2-300^2}}{500}(10) = 8 \text{ ft./sec.}$$

10. If *x* and *y* are the numbers, $xy=2$. So, $y=\frac{2}{x}$.

We want to minimize $x+y = x+\frac{2}{x}$. At a critical point, $\frac{d}{dx}\left(x+\frac{2}{x}\right) = 1-\frac{2}{x^2} = 0$. So, $x^2=2$, $x=\pm\sqrt{2}$. Since $x>0$, $x=+\sqrt{2}$ and $y=\frac{2}{\sqrt{2}}=\sqrt{2}$. Their sum is $2\sqrt{2}$.