3.1 (Continued)

Ex. a) Find the rate of change of

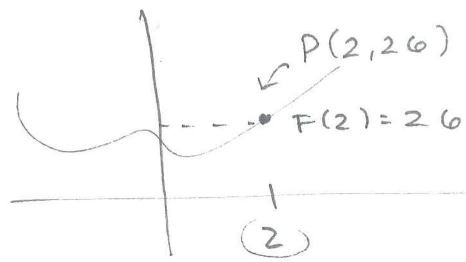
$F(t) = 2t^4 - 3t$ at $t = 2$. $\quad F(2) = 26$

a typical secant has slope from $(2, 26)$ to $(b, F(b))$ $\quad \dfrac{F(b) - F(2)}{b-2} = \dfrac{2b^4 - 3b - 26}{b-2}$

$= 2b^3 + 4b^2 + 8b + 13$

approaches 61 when b approaches 2.

$$
\begin{array}{r}
2b^3 + 4b^2 + 8b + 13 \\
b - 2 \;\overline{)\; 2b^4 - 3b - 26} \\
-(2b^4 - 4b^3) \\
\hline
4b^3 - 3b - 26 \\
-(4b^3 - 8b^2) \\
\hline
8b^2 - 3b - 26 \\
-(8b^2 - 16b) \\
\hline
13b - 26 \\
-(13b - 26) \\
\hline
0
\end{array}
$$



b) Find the equation of the tangent line to F @ $t = 2$.

Equation of a line with slope m, through $P(t_0, y_0)$

is $\quad y - y_0 = m(x - t_0)$

Here $\quad m = 61 \quad P(2, 26)$

Tangent: $\quad y - 26 = 61(x - 2)$

$y = 61x - 122 + 26$

$\boxed{y = 61x - 96}$

2. Find an equation to the line tangent to the graph of $F(t) = \frac{1}{t^2}$ at $P(2, 1/4)$

the slope of a secant from $P(2, 1/4)$ to any point $(b, 1/b^2)$

is
$$\frac{F(b) - F(2)}{b-2} = \frac{\frac{4 \cdot 1}{4b^2} - \frac{b^2}{4b^2}}{b-2} = \frac{\frac{4-b^2}{4b^2}}{\frac{b-2}{1}} = \frac{4-b^2}{4b^2} \cdot \frac{1}{b-2}$$

$$= \frac{-(\cancel{-2+b})(2+b)}{4b^2(\cancel{b-2})} = -\frac{(b+2)}{4b^2}$$ as b approaches 2,

the slope of the secants approach $-\frac{1}{4}$ ← slope at the tangent

Eqn: $m = -\frac{1}{4}$ and $P(2, 1/4)$

$$y - 1/4 = -\frac{1}{4}(t-2)$$

$$y = -\frac{1}{4}t + \frac{2}{4} + \frac{1}{4} \Rightarrow \boxed{y = -\frac{1}{4}t + \frac{3}{4}}$$

3. At what rate is the function $F(t) = \sqrt[3]{t}$ increasing at $t = 8$?

$$\frac{F(b) - F(8)}{b - 8} = \frac{\sqrt[3]{b} - 2}{b - 8} = \frac{\sqrt[3]{b} - 2}{(\sqrt[3]{b})^3 - 2^3}$$ (diff of cubes)

$$= \frac{\cancel{(\sqrt[3]{b} - 2)}}{\cancel{(\sqrt[3]{b} - 2)}(b^{2/3} + 2b^{1/3} + 4)}$$

↑ as b approaches 8

the slope approaches: $\frac{1}{12}$

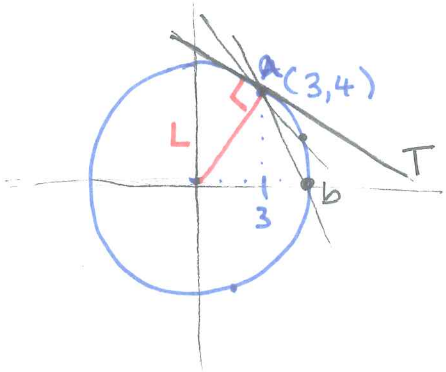
For the Equation we need the point @ $t = 8$

$P(2, F(2)) = P(8, 2)$, $m = \frac{1}{12}$

$$y - 2 = \frac{1}{12}(t - 8)$$

$$y = \frac{1}{12}t - \frac{8}{12} + 2 \rightarrow \boxed{y = \frac{1}{12}t + \frac{4}{3}}$$

4. What slope do the secants in the fig. approach.



$x^2 + y^2 = 25$

Line L has slope $\frac{4}{3}$ so that

The line T has slope $-\frac{3}{4}$

L and T are perpendicular!

$F = y = \sqrt{25 - x^2}$

$$\frac{F(b) - F(3)}{b-3} = \frac{\sqrt{25-b^2} - 4}{b-3} \cdot \frac{\sqrt{25-b^2}+4}{\sqrt{25-b^2}+4}$$

$$= \frac{(25-b^2) - 16}{(b-3)\sqrt{25-b^2}+4} = \frac{9-b^2}{(b-3)\sqrt{25-b^2}+4}$$

$$= \frac{-\cancel{(3-b)}(3+b)}{\cancel{(b-3)}\sqrt{25-b^2}+4}$$

$$= \frac{-(b+3)}{\sqrt{25-b^2}+4} = -\frac{3+3}{4+4} = -\frac{6}{8} = -\frac{3}{4}$$

when b approaches 3, the slope approaches $-\frac{3}{4}$