

Notes to Volunteer:

What to do: 1. set up rugs and cardboard bridges in the configuration shown in the diagram if that isn't already done.

2. Read the display and try the activities yourself.

3. Read this note and get familiar with the solution and with the mathematics.

4. Think about how you will guide kids through the activity.

The mathematics.

The answer to the Koenigsberg Bridge problem is that it is impossible to walk each bridge exactly once.

Vocabulary. The diagram is a graph, the dots are vertices, and the lines are edges.

On this diagram each dot represents one of the locations connected by the bridges and each line represents a bridge. You can count the number of edges which meet at each vertex. There are 5 at one vertex (Kneiphof Island) and 3 at each of the other vertices.

Here's what Euler proved: If more than 2 vertices on a graph have an odd number of edges meeting there then it is impossible trace each edge exactly once.

There are 3 other questions on the display, and the answer to each one is yes. 1).

Remove Shopkeepers Bridge---> the number of edges meeting at Kneiphof is now 4 and the number of edges meeting at the upper bank of town is 2. That's only 2 odd vertices.

2) Remove Green Bridge instead and Kneiphof goes down to 4 and lower bank goes down to 2. 3) Build a new bridge: now there are 6 meeting at Kneiphof and 4 meeting at upper bank. Only two odd vertices.

\*Guidance for students: You'll probably want to encourage them to walk the bridges & see if they can do it or whether they think it's impossible.

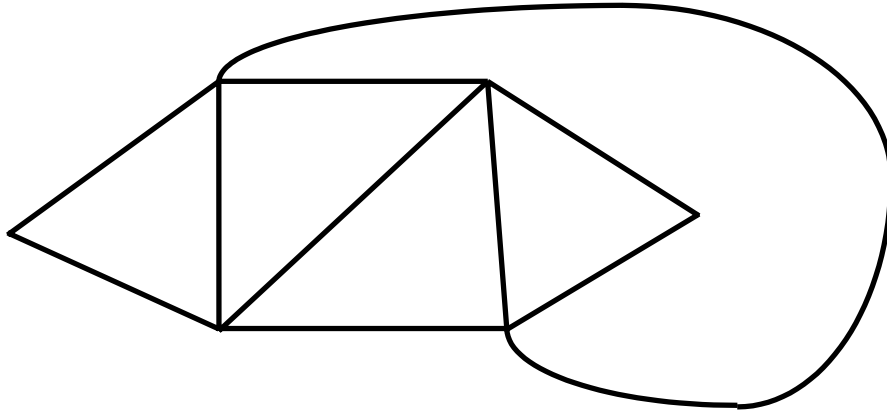
They can try removing one bridge at a time or adding a bridge--you might want to let them know that that's ok.

How to use the diagram. They can put the dots on a piece of scrap paper and see if they can duplicate the diagram without retracing their lines or picking up their pencils. They can try also with the variations suggested.

It is unlikely that most kids will generalize to this rule for deciding whether or not the bridges can be walked. However, don't help them too soon--let them try and think about it first!

More detail: the theory in a nutshell.

Suppose you want to trace each edge exactly once AND land back where you started.



You can quickly verify that it is possible on this graph. The key is that each vertex has an even number of edges meeting there--which means that there are as many edges in as there are out. You can go all the way around, tracing each leg, and wind up back where you started.

Now suppose you pull out one of the edges. That leaves you with 2 vertices which have an odd number of edges meeting there. You can, of course, still trace all the paths in the picture but you're missing the leg that gets you from where you wind up to where you started.

If you pull one more edge out you have an odd number of edges meeting at each of 4 vertices. Now it is impossible even to trace all of the edges without picking up your pencil.