The Simplex Method

Algorithm, Example, and TI-83 / 84 Instructions

Before you start, set up your simplex tableau. Be sure to label all of the columns and label the basic variables with markers to the left of the first column (see the sample problem below for the initial label setup). If you are using a calculator, enter your tableau into your calculator as a matrix (obviously, you cannot enter the labels, but you should have them written down on your paper copy of the tableau).

Example Problem

Maximize

$P = 3x_1 + 5x_2$

Subject to

$x_1 \leq 4$

$2x_2 \leq 12$

$3x_1 + 2x_2 \leq 18$

$x_1 \geq 0, x_2 \geq 0$

Initial Tableau

	x_1	x_2	s_1	s_2	s_3	P	
s_1	1	0	1	0	0	0	4
s_2	0	2	0	1	0	0	12
s_3	3	2	0	0	1	0	18
P	-3	-5	0	0	0	1	0

Initial basic feasible solution (BFS):

$x_1 = 0, \; x_2 = 0, \; s_1 = 4, \; s_2 = 12, \; s_3 = 18, \; P = 0$

Step 1.

Check the bottom row for negative indicators. Are there any?

Yes: Go to step 2.

No: You are done - **optimal solution has been reached**.

Example.

We check the bottom row of the tableau for negative values:

	x_1	x_2	s_1	s_2	s_3	P	
s_1	1	0	1	0	0	0	4
s_2	0	2	0	1	0	0	12
s_3	3	2	0	0	1	0	18
P	-3	-5	0	0	0	1	0

Notice we have -3 in column 1 and -5 in column 2. Therefore, we are not at the optimal solution yet, so we go on to step 2.

Here is an example of a tableau with no negative indicators; the optimal solution has been reached for this problem:

	x_1	x_2	s_1	s_2	P	
s_1	1	0	1	-9	0	4
x_2	3	1	0	1	0	12
P	3	0	0	7	1	80

Step 2.
Select the most negative number in the bottom row. The column this number is in is the **pivot column**. The **entering variable** is the variable that corresponds to this column (check the label at the top of the column).

Example.
The most negative value in the bottom row is -5, so our pivot column is column 2. The entering variable is x_2, since this column corresponds to x_2 (check the label above the column).

	x_1	x_2	s_1	s_2	s_3	P	
s_1	1	0	1	0	0	0	4
s_2	0	2	0	1	0	0	12
s_3	3	2	0	0	1	0	18
P	-3	-5	0	0	0	1	0

Step 3.
Look at the entries in the pivot column above the negative number. Are there any positive values (values greater than 0)?
No: You are done - **no solution (unbounded feasible region)**.
Yes: Go to step 4.

Example.
In our example, we have the values 0, 2, and 2. One of these is 0, but the other two are positive, so we can go to step 4.

	x_1	x_2	s_1	s_2	s_3	P	
s_1	1	0	1	0	0	0	4
s_2	0	2	0	1	0	0	12
s_3	3	2	0	0	1	0	18
P	-3	-5	0	0	0	1	0

As an aside, here is an example of a tableau where the values in the pivot column are all either 0 or negative (no positive numbers in the pivot column). In this case, there is no solution, so we would stop:

	x_1	x_2	s_1	s_2	s_3	P	
s_1	1	0	1	0	0	0	4
s_2	0	-2	0	1	0	0	12
s_3	3	-2	0	0	1	0	18
P	-3	-5	0	0	0	1	0

Step 4.
For each row that has a non-negative entry in the pivot column, divide the number in the far-right column of the tableau by the number in the pivot column. The row that has the smallest result here is your **pivot row**, and the **exiting variable** is the one that is basic in this row (check the label to the left of the row).

Example.
Our tableau has two rows with positive values in the pivot column, so we need to compute the ratios described. For row 2, we compute 12/2 = 6. For row 3, we get 18/2 = 9. The smallest value is 6, so row 2 is our pivot row. Checking our labels, s_2 is basic on row 2, so s_2 is our exiting variable.

	x_1	x_2	s_1	s_2	s_3	P		
s_1	1	0	1	0	0	0	4	(Entry in pivot column is 0, so skip this row)
s_2	0	2	0	1	0	0	12	12/2 = 6
s_3	3	2	0	0	1	0	18	18/2 = 9
P	-3	-5	0	0	0	1	0	

Step 5.
The pivot is the element that is located where the pivot column and pivot row intersect. Is the pivot element a one (1)?

Yes: Go to step 7.
No: Go to step 6.

Example.
In our example, our pivot is a 2, so we will need to turn it to a 1 in step 6. The pivot row and pivot column have been highlighted to emphasize this:

	x_1	x_2	s_1	s_2	s_3	P	
s_1	1	0	1	0	0	0	4
s_2	0	2	0	1	0	0	12
s_3	3	2	0	0	1	0	18
P	-3	-5	0	0	0	1	0

Here is a tableau where the pivot is a 1 (you should go through steps 1-4 to verify that the pivot was correctly selected), so we would skip to step 7 in this case:

	x_1	x_2	s_1	s_2	P	
s_1	1	0	1	-9	0	4
x_2	3	1	0	1	0	21
P	-3	0	0	7	1	80

In this case, the entering variable is x_1, and the exiting variable is s_1.

Step 6.
The pivot is not a one (1), so you need to do a matrix row operation to turn the pivot into a 1. This is done by dividing the entire row by the value of the pivot. In general, if the pivot is given by M and your pivot row is row K, you need to perform the row operation $(1/M)R_K \rightarrow R_K$.

On a TI-83+ or TI-84, the command for this is `*row()`, which is under `2nd` > `Matrix` > `Math`. The format of the command is

`*row(1/M,[A],K)`

or, more generally,

`*row( value you are multiplying by, matrix you are using, pivot row )`

NOTE: The row operation is **not** saved automatically, so **you need to save your matrix manually**. After doing your row operation, press the `STO→` button, then `2nd` > `Matrix`, then select the matrix you are using (your calculator should say `Ans→[A]`), then press `Enter`. This will update your matrix.

Example.
Our pivot is 2, and our pivot row is row 2. Therefore, we need to compute $(1/2)R_2 \rightarrow R_2$. On the calculator, if my tableau is stored to matrix `[A]`, I would type `*row(1/2,[A],2)`, verify that my pivot was changed to a 1, then press `STO→` and grab `[A]` out of the matrix names menu, then `Enter` to save my work.

	x_1	x_2	s_1	s_2	s_3	P	
s_1	1	0	1	0	0	0	4
s_2	0	2	0	1	0	0	12
s_3	3	2	0	0	1	0	18
P	-3	-5	0	0	0	1	0

→

	x_1	x_2	s_1	s_2	s_3	P	
s_1	1	0	1	0	0	0	4
s_2	0	1	0	1/2	0	0	6
s_3	3	2	0	0	1	0	18
P	-3	-5	0	0	0	1	0

Step 7.
Your pivot is now a 1. You need to perform row reduction on the pivot column: the pivot stays as a 1, and the rest of the entries need to become zeros. Generally, if your pivot row is row K, and you need to turn the value M in column L to zero, you perform the row operation $(-M)R_K + R_L \rightarrow R_L$. On a TI-83+ or TI-84, the command for this is `*row+()`, which is found under `2nd` > `Matrix` > `Math`. The format of the command you will type is

`*row+(-M,[A],K,L)`

or, more generally,

`*row+( value, matrix, pivot row, row that you want to change )`

NOTE: Make sure that **after each individual row operation** you save the matrix with the STO→ button, as described in step 6.

Example.

We need to cancel the 2 in row 3 and the -5 in row 4 of our tableau. Row 1 already has a 0 in the pivot column, so we do not need to do anything to row 1.

	x_1	x_2	s_1	s_2	s_3	P	
s_1	1	0	1	0	0	0	4
s_2	0	1	0	1/2	0	0	6
s_3	3	2	0	0	1	0	18
P	-3	-5	0	0	0	1	0

First, we compute $-2R_2 + R_3 \rightarrow R_3$. On the calculator: `*row+(-2,[A],2,3)`. We get:

	x_1	x_2	s_1	s_2	s_3	P	
s_1	1	0	1	0	0	0	4
s_2	0	1	0	1/2	0	0	6
s_3	3	0	0	-1	1	0	6
P	-3	-5	0	0	0	1	0

Then, we compute $5R_2 + R_4 \rightarrow R_4$. On the calculator: `*row+(5,[A],2,4)`. We get:

	x_1	x_2	s_1	s_2	s_3	P	
s_1	1	0	1	0	0	0	4
s_2	0	1	0	1/2	0	0	6
s_3	3	0	0	-1	1	0	6
P	-3	0	0	5/2	0	1	30

This completes the pivot operation: our pivot column has a 1 in row 2, and 0 elsewhere.

Step 8.

Once you have completed the pivot operation, update the basic variable label on the left side of your matrix. Change the label on the pivot row (which was the exiting variable) to the entering variable (determined in step 2). You should also update your basic feasible solution at this time.

Example.

During our pivot operation, x_2 entered the basis (x_2 was the entering variable), and s_2 left the basis (s_2 was the exiting variable). We need to update our basic variable labels to reflect this change by replacing s_2 with x_2:

	x_1	x_2	s_1	s_2	s_3	P	
s_1	1	0	1	0	0	0	4
x_2	0	1	0	1/2	0	0	6
s_3	3	0	0	-1	1	0	6
P	-3	0	0	5/2	0	1	30

Remember: basic variables are indicated by the far left column of labels. If a variable is listed there, it has the value in the far right column on the same row. The variables that are not listed in the far left column of labels are all equal to 0.

	x_1	x_2	s_1	s_2	s_3	P	
s_1	1	0	1	0	0	0	4
x_2	0	1	0	1/2	0	0	6
s_3	3	0	0	-1	1	0	6
P	-3	0	0	5/2	0	1	30

With this in mind, we can see that our current basic feasible solution is $x_1 = 0$, $x_2 = 6$, $s_1 = 4$, $s_2 = 0$, $s_3 = 6$, $P = 30$. (We have $x_1 = 0$ and $s_2 = 0$ since these values do not appear in the far left column.)

Step 9.
Go back to step 1 and do the steps again. You will keep looping through these steps until you either get to an optimal solution in step 1, or get to a no solution situation in step 3.

Example.
Let's finish our example. First, we identify the pivot (steps 1-4):

	x_1	x_2	s_1	s_2	s_3	P		
s_1	1	0	1	0	0	0	4	4/1 = 4
x_2	0	1	0	1/2	0	0	6	(Entry is a 0 so we skip it)
s_3	3	0	0	-1	1	0	6	6/3 = 2
P	-3	0	0	5/2	0	1	30	

Our pivot is a 3, so we need to divide row 3 by 3 (steps 5-6):

	x_1	x_2	s_1	s_2	s_3	P	
s_1	1	0	1	0	0	0	4
x_2	0	1	0	1/2	0	0	6
s_3	1	0	0	-1/3	1/3	0	2
P	-3	0	0	5/2	0	1	30

Next, we have to do the pivot operation to get zeros in rows 1 and 4, and update our far-left column (steps 7-8):

	x_1	x_2	s_1	s_2	s_3	P	
s_1	0	0	1	1/3	-1/3	0	2
x_2	0	1	0	1/2	0	0	6
x_1	1	0	0	-1/3	1/3	0	2
P	0	0	0	3/2	1	1	36

Without shading, our tableau at the end of this pivot operation is given by

	x_1	x_2	s_1	s_2	s_3	P	
s_1	0	0	1	1/3	-1/3	0	2
x_2	0	1	0	1/2	0	0	6
x_1	1	0	0	-1/3	1/3	0	2
P	0	0	0	3/2	1	1	36

We have the basic feasible solution $x_1 = 2$, $x_2 = 6$, $s_1 = 2$, $s_2 = 0$, $s_3 = 0$, $P = 36$.

We have finished the pivot operation, so we go back to step 1 and check the bottom row. There are no more negative indicators in the bottom row, so we have reached an optimal solution and are done with the simplex method.

Our variables are x_1 and x_2, and our objective function that we wanted to maximize is P, so the optimal solution to our sample problem is $x_1 = 2$, $x_2 = 6$, and $P = 36$.