1. Please answer the following questions (3 points each)  
_____  
(a) Let $F_N$ be the $N$th Fibonacci number. Write down the recursive rule satisfied by the Fibonacci numbers.  
\[ F_N = F_{N-1} + F_{N-2} \]  
(b) Given $F_{13} = 233$, $F_{14} = 377$. What is the value of $F_{12}$?  
\[ 144 \]  
(c) What is the exact value of the golden ratio?  
\[ \frac{\sqrt{5} + 1}{2} \]  
(d) Assume linear growth model. If the initial population is $P_0$ and the common difference is $d$. What is the population of the $N$’s generation?  
\[ P_0 + d \times N \]  
(e) What types of symmetry does a square have?  
\[ \text{Diagonal} \]  

2. (2pt each) Determine whether the following statements are true or false.  

(a) $\sqrt{5}$ is an irrational number.  
\[ \text{Yes} \]  
(b) The sequence of Fibonacci numbers is a geometric sequence?  
\[ \text{No} \]  
(c) Any two squares are similar.  
\[ \text{Yes} \]  
(d) The line graph of an exponential growth model is a straight line.  
\[ \text{Yes} \]  
(e) $2, 5, 8, 10, \ldots$ can be the first few terms of an arithmetic sequence.  
\[ \text{No} \]  
(f) An improper rigid motion will reverse left-right orientation but preserve clockwise-counter clockwise orientation.  
\[ \text{Yes} \]  
(g) A reflection is completely determined by the axis of reflection.  
\[ \text{Yes} \]  
(h) A vector is a line segment with direction.  
\[ \text{Yes} \]  
(i) Given any two different points $P$ and $P'$ in the plane. There are only one rotation which moves $P$ to $P'$.  
\[ \text{No} \]  
(j) The type of symmetry for letter “A” is $\text{D}_2$.  
\[ \text{No} \]  

3. (5 pts each) Multiply choice questions. Please choose one of the following answers.  

(a) The value of $1 + 3 + 5 + \cdots + 101$ is $\frac{102 \times 51}{2}$.  
\[ \text{D} \]  

(b) The value of $1 + 3 + 3^2 + \cdots + 3^{99}$ is $\frac{3^{100} - 1}{3 - 1}$, $\frac{3^{100} - 1}{3 - 1}$, $\frac{3^{101} - 1}{3 - 1}$.  
\[ \text{C} \]  

(c) Consider a population that grows according to a linear growth model. If the initial population is $P_0 = 5$, and the common difference is $d = 2$. What is $P_{10}$?  
\[ \text{A} \]  

(d) Consider a population that grows according to a linear growth model. If $P_0 = 6$ and $P_{10} = 26$, what is the common difference $d$?  
\[ \text{B} \]
(e) $\sqrt{2} + 1$ is a solution of which of the quadratic equation?
A). $x^2 - 2x + (1 - a^2)$; B). $x^2 - 2x + (1 + a^2)$; C). $x^2 + 2x + (1 - a^2)$; D). $x^2 + 2x + (1 + a^2)$

(f) Rotation clockwise by $90^\circ$ is equivalent to
A). Clockwise by $390^\circ$; B). Counterclockwise by $270^\circ$; C). Counterclockwise by $540^\circ$; D). Counterclockwise by $\pi/2$

(g) Find the value of $x$ so that the shaded triangle is a gnomon to the white triangle $ABC$.

(Please use the picture in the Textbook P384 for exercise 44).

(h) Find the value of $c$ so that the shaded rectangle is a gnomon to the white rectangle with size 3 and 9.
A). 9, B). 8, C). 7, D). $\frac{9}{2}$

(Please use the picture in the Textbook P383 for exercise 37).

4. Answer the following questions use the figure on page 460 for exercise 23.

(a) Find the image of $B$ under the reflection with the given axis?
(b) Find the image of $B$ under a $90^\circ$ clockwise rotation with rotocenter $A$.
(c) Find the image of the quadrilateral $ABCD$ under the glide reflection specified by the vector $v$ and axis $l$?

See answers in the book.

5. Deposit $100 in a saving account that pays 10% annual interest.

(a) (2pts)In the plan A, if the interest is compounded annually, how much money in the bank after three year? $\text{100} \times (1.1)^3$ = $133.10$
(b) (2pts)In the plan B, if the interest is compounded quarterly (four times a year), how much money in the bank after one year? $\text{100} \times (1 + \frac{0.1}{4})^4$ = $110.38$
(c) What is the annual yield of the plan B?

6.

(a) List all plane rigid motions which are proper.
(b) List all plane rigid motions which are improper.
MATHEMATICS 105
Quiz 5 F05

Print name
Last name
First name

(In this quiz, please show your calculation to get full credit.)

1. (3pt) Consider a population that grows according to a linear growth model. If the initial population is \( P_0 = 5 \), and the common difference is \( d = 2 \). What is \( P_{20} \)?

\[
P_{20} = P_0 + d \cdot n
= 5 + 2 \cdot 20
= 45
\]

2. (3pts) Consider a population that grows according to a linear growth model. If \( P_0 = 6 \) and \( P_{10} = 26 \), what is the common difference \( d \)?

\[
2b = P_0 + 6 + 10d
\]
\[
z = d
\]

3. Deposit $100 in a saving account that pays 8% annual interest.

(a) (2pts) If the interest is compounded annually, how much money in the bank after two years?

\[
100 \times (1.08)^2
\]
\[= 116.64\]

(b) (2pts) If the interest is compounded semi-annually (twice a year), how much money in the bank after one year?

\[
100 \times \left(1 + \frac{0.08}{2}\right)^2 = 108.16
\]

(Bonus) (2pt) Use the geometric sum formula to calculate \( 12 + 15 + 18 + 21 + \cdots + 102 \).

\[
d = 3
\]
\[
P_N = 12 + (N + 1) \times 3
\]
\[
(N + 1) \times 3 = 90
\]
\[
N + 1 = 30
\]
\[
N = 29
\]
\[
= 1767.
\]
1. (3pt) Consider a population that grows according to a linear growth model. If the initial population is \( P_0 = 5 \), and the common difference is \( d = 2 \). What is \( P_{20} \)?

\[
P_{20} = P_0 + 20d
= 5 + 2 \times 20
= 45
\]

2. (3pts) Consider a population that grows according to a linear growth model. If \( P_0 = 6 \) and \( P_{10} = 26 \), what is the common difference \( d \)?

\[
2b = P_{10} + 6 + 10d
2 = d
\]

3. Deposit $100 in a saving account that pays 8% annual interest.

(a) (2pts) If the interest is compounded annually, how much money in the bank after two years?

(b) (2pts) If the interest is compounded semi-annually (twice a year), how much money in the bank after one year?

(a) \[
100 \times (1.08)^2
\]

(b) \[
100 \times (1 + \frac{0.08}{2})^2 = 108.16
\]

(Bonus) (2pt) Use the geometric sum formula to calculate \( 12 + 15 + 18 + 21 + \cdots + 102 \).

\[
P_{N+1} = 12 + (N+1) \times 3
\]

\[
(N-1) \times 3 = 90
N - 1 = 30
N = 31
\]

\[
\frac{0.08 \times (0.08)^3}{2}
= 1767.
\]
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1. (2pts) List all plane rigid motions which are proper. (translation, rotation)
2. (2pts) List all plane rigid motions which are improper. (reflection, glide reflection)
3. (1pt each) Determine each of the following statements is true or false.

   (a) An improper rigid motion will reverse left-right and clockwise-counterclockwise orientation. **Yes**
   (b) A reflection is completely determined by the axis of reflection. **Yes**
   (c) A rotation is completely determined by the angle of rotation. **No**
   (d) A vector is a line segment with direction. **Yes**
   (e) A glide reflection is a reflection followed by any arbitrary translation. **No**
   (f) Given any two different points $P$ and $P'$ in the plane. There are only one rotation which moves $P$ to $P'$. **Infinitely many rotations**