Homework on Fourier transformation

Let $\mathcal{L}$ be the space of infinitely differentiable functions $f : \mathbb{R} \to \mathbb{C}$ which decreases at infinitely faster than any negative power functions, i.e. $|x|^N f(x) \to a$ as $x \to \pm \infty$ for all $N$. For any function $f \in \mathcal{L}$ we define its Fourier transform $\hat{f}$ by

$$\hat{f}(y) = \int_{-\infty}^{\infty} e^{-2\pi ixy} f(x) dx.$$  (1)

(1) Prove that If $f(x) = e^{-\pi x^2}$, then $\hat{f} = f$.

(2) Prove the Poisson summation formula which says if $g \in \mathcal{L}$, then

$$\sum_{m=-\infty}^{\infty} g(m) = \sum_{m=-\infty}^{\infty} \hat{g}(m).$$  (2)

(3) Prove that if we define the theta function as

$$\theta(t) = \sum_{n=-\infty}^{\infty} e^{-\pi tn^2},$$

then it satisfies

$$\theta(t) = \frac{1}{\sqrt{t}} \theta(1/t)$$  (3)