Here are some useful equations:

<table>
<thead>
<tr>
<th>Type</th>
<th>Recursive</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear growth</td>
<td>( P_N = P_{N-1} + d )</td>
<td>( d ) is the common difference</td>
</tr>
<tr>
<td>Exponential growth</td>
<td>( P_N = rP_{N-1} )</td>
<td>( r ) is the common ratio</td>
</tr>
<tr>
<td>Arithmetic sum</td>
<td>( A_0 + A_1 + \cdots + A_{N-1} = \frac{(A_0 + A_{N-1}) \times N}{2} )</td>
<td></td>
</tr>
<tr>
<td>Geometric sum</td>
<td>( a + ar + ar^2 + \cdots + ar^{N-1} = \frac{a(r^N - 1)}{r-1} )</td>
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</table>

1. Please answer the following questions (3 points each)  

(a) The complete graph with \( N \) vertices has how many Hamilton circuits starting at a fixed vertex? ( )

(b) A project consisting of \( M \) tasks, the number of possible priority lists is ( )

(c) Let \( F_N \) denote the \( N \)-th Fibonacci number. Given \( F_{13} = 233, F_{14} = 377 \). What is the value of \( F_{12} \)? ( )

(d) Assume linear growth model. If the initial population is 5 and the common difference is 4. What is the population of the 10’s generation? ( )

(e) What type of symmetry does a square have? ( )

(f) Toss a coin 10 times and observe on each toss whether it lands heads or tails. How big is the sample space? ( )

(g) There are 6 candidates are running in an election. How many possible ways to choose among them a president and a vice president? Please write down the exact solution. ( )

(h) Cafe Sun offers 6 different kinds of bagels and 4 different kinds of cream cheese. How many ways to get one bagel with a cream cheese? Please write down the exact solution. ( )

(i) Consider the sample space \( S = \{o_1, o_2, o_3, \} \). Suppose you are given \( Pr(o_1) + Pr(o_2) = 3Pr(o_3) \). If \( Pr(o_1) = 0.20 \). Find \( Pr(o_2) \). ( )

(j) Alex and Bob would like to share a chocolate-strawberry cake which they paid for $12 by using the divider-chooser method. If Alex is the divider, what is the value of the share he will get in the end to him? ( )
2. Determine whether the following statements are true or false.  
Please use A for True and B for False.  

(a) A connected graph with only two odd vertices has an Euler circuit. ( )  
(b) A connected graph without loops is a tree. ( )  
(c) Kruskal’s algorithm in finding a minimal spending tree is an approximate algorithm. ( )  
(d) Any two isosceles triangles are always similar. ( )  
(e) The golden ratio is a solution of the quadratic equation $x^2 - x = 1$. ( )  
(f) 2, 6, 18, 27, … may be the first few terms of a geometric series. ( )  
(g) Glide reflection is an improper rigid motion. ( )  
(h) A probability assignment always assumes that the probability of the total sample space is 1. ( )  
(i) In the fair share game, a particular share may be fair to one player and may not be fair to another player. ( )  

3. Multiply choice questions. Please choose ONE of the following answers.  

(a) Consider a population that grows according to a linear growth model. If $P_0 = 6$ and $P_{10} = 36$, what is the common difference $d$? (A). 1; B). 2; C). 3; D). 4  
(b) In an arithmetic sequence, if $a_0 = 1, a_1 = 4$, then $a_{100} =$ (A). 298; B). 301; C). 397; D). 401  
(c) To write down all possible priority lists for scheduling 5 task $A, B, C, D, E$. How many possible priority list will be compatible with the following project digraph? (A). $5!/2$; B. 5!; C). $2C_5$; D). $2P_5$
(d) **(Use the picture on p.383 for problem 40.)** Find the value of \( x \) so that the shaded figure is a gnomon to the white rectangle.

A). 1 ; B). 3/2 ; C). 2 ; D). 5/2

(e) Rotation clockwise by 30° is equivalent to

A). Counterclockwise 510°; B). Counterclockwise by 790°; C). Counterclockwise by 690°; D). Counterclockwise by 270°.

(f) How many ways to choose a committee of 3 members among 6 candidates where all committee members have equal standing?


(g) To open an account in US bank requires a pin consisting of 4 digits each digit choose from (0 through 9).

How many possible different pins there are?


(h) Alex buys a chocolate-strawberry cake with half chocolate and half strawberry flavors for $12.

Alex values chocolate 2 times as he values strawberry.

What is the value of the strawberry half of the cake to Alex?

A). 3; B). 4; C). 8; D). 9

(i) Paul and Karli want to divide a chocolate-strawberry mousse cake by using the divider-chooser method. Paul values chocolate three times as much as he values strawberry. If Raul is the divider, which of the following cuts are consistent with Raul’s value system?

**(Use the graphs on p. 115 for problem 10)**

A). Cut 1 alone; B). Cut 1 and 3 only; C). Cut 1 and 5 only; D). Cut 1, 4, and 5
4. Given the weighted graph as below.

(a) Is this weighted graph complete? 
(b) Does this graph has an Euler circuit? 
(c) How many Hamilton circuits this graph has? 
(d) Apply the cheapest-link algorithm to find a Hamilton circuit. (5 points) 
(e) Apply the Kruskal’s algorithm to get a minimal spanning tree for this graph. Please draw your minimal spanning tree below. (5 point)
5. (Use the graph on p. 460 for problem 24) Please answer the following questions. 10 pt

   (a) (2pts) Find the image of \( A \) under a 90° rotation clockwise with rotocenter \( B \).

   (b) (2pts) Find image of \( B \) under the translation given by the vector \( v \).

   (c) (2pts) Find the image of \( B \) under the reflection with the given Axis.

   (d) (4pts) Find image of the shaded area under a glide reflection given by vector \( v \) and the given Axis.

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6. Consider the random experience of rolling a pair of honest dice. 20 pt

   (a) Observe the number on each die. How big is the sample space. ( )

   (b) Observe the total of the two dice. How big is the sample space? ( )

   (c) What is the probability of getting a total of exactly 2. ( )

   (d) Let \( E \) be the even describing “roll a total of 3 or less”. Write out the event by describing it as a set. ( )

   (e) Find the probability of event \( E \). ( )
7. **(Use the picture on p.116 for problem 12)** Jamie and Mo want to divide an orange-pineapple cake using the divider-chooser method. Jamie values orange four times as much as he values pineapple.

(a) If they decide by tossing an honest coin to decide who will be the divider. What is the probability of Mo being the divider? 

(b) Suppose Mo is the divider and cuts the cake as shown in p. 116. What percent of the value of the cake is the pineapple half in Mo’s eyes? 

(c) What percent of the value of the cake is each piece in the the picture in Jamie’s eyes? 

(d) Describe the final division of the cake.