1. • Find \( Aut(\mathbb{Z}_{12}) \).
   • Is \( \mathbb{Z}_3 \oplus \mathbb{Z}_9 \) isomorphic to \( \mathbb{Z}_{27} \)? Why?
   • Is \( \mathbb{Z}_3 \oplus \mathbb{Z}_5 \) isomorphic to \( \mathbb{Z}_{15} \)? Why?
   • Find all subgroups of order 4 in \( \mathbb{Z}_4 \oplus \mathbb{Z}_4 \).  \[ 20 \text{ pt} \]

2. Show that \( U(8) \) is not isomorphic to \( U(10) \).  \[ 8 \text{ pt} \]

3. Let \( G \) be a group. Prove that the mapping \( \alpha(g) = g^{-1} \) is an automorphism if and only if \( G \) is abelian.  \[ 10 \text{ pt} \]

4. Suppose that \( G \) is a finite group of order \( n \) and \( m \) is relatively prime to \( n \). If \( g \in G \) and \( g^m = e \), prove that \( g = e \).  \[ 10 \text{ pt} \]

5. Let \( H = \{ \epsilon, (12) \} \) be a subgroup of \( S_3 \).
   • List all the left cosets of \( H \) in \( S_3 \).
   • Find the orbits of 1 under \( H \).
   • Find the stabilizers of 2 in \( S_3 \).
   • Is \( H \) normal in \( S_3 \)? Please explain why.  \[ 16 \text{ pt} \]

6. Let \( G = S_4 \). For every \( \sigma \in S_4 \), define
   \[
   sgn(\sigma) = \begin{cases} 
   +1 & \text{if} \sigma \text{ is an even permutation;} \\
   -1 & \text{if} \sigma \text{ is an odd permutation.}
   \end{cases}
   \]
   (a) Prove that \( sgn \) is a homomorphism from \( S_4 \) to the multiplicative group \( \{+1, -1\} \).
   (b) What is the kernel?
   (c) Prove that \( A_4 \) is normal in \( S_4 \).
   (d) List elements of the quotient group \( S_4/A_4 \).  \[ 20 \text{ pt} \]

7. Prove that any abelian group of order 45 has an element of order 15.  \[ 10 \text{ pt} \]

8. How many abelian groups there are of order 15?  \[ 6 \text{ pt} \]