1. Suppose that a finite group $G$ is generated by two elements $a$ and $b$ (that is, every element of the group can be expressed as some product of $a$’s and $b$’s). Given that $a^3 = e = b^2$ and $ab = ba^2$.
   - Construct the Cayley table for the group.
   - Find the center $Z(G)$ of $G$ and the centralizer $C(a)$ of $a$ in $G$.

2. (a) List all elements of $Z_{900}$ of order 10.
   (b) List all generators of $U(20)$.

3. Let $G = \{a + b\sqrt{2}\}$, where $a$ and $b$ are rational numbers not both 0. Prove that $G$ is a group under ordinary multiplication.

4. Prove that a group of order 5 is cyclic.

5. Let $\alpha = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 5 & 4 & 6 & 3 \end{bmatrix}$, $\beta = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 1 & 2 & 3 & 5 & 4 \end{bmatrix}$
   - Find $|\alpha|$, $|\beta|$.
   - Write $\alpha\beta$ as a product of disjoint cycles.
   - Find $|\alpha\beta|$.
   - Is $\alpha\beta$ an even or odd permutation?