1. Please answer the following questions (3 points each)  

(a) If a connected graph has only two odd vertices, does it have any Euler path? ( )  
(b) Who first solved Fermat’s problem for finding the Steiner point for a set of three points? ( )  
(c) Does the complete graph with 5 vertices has an Euler path? ( )  
(d) Is the shortest link algorithm an inefficient algorithm? ( )  
(e) Is the closest neighbor algorithm an optimal algorithm? ( )  
(f) What is the degree of every vertex in a complete graph with $N$ vertices? ( )  
(g) How many edges does the complete graph with $N$ vertices have? ( )  
(h) How many edges does a tree with 10 vertices have? ( )

2. Let $G$ be a graph with no loops or multiply edges. Please choose from the following three options the one that best applies (2 points each)  

(I) The graph $G$ is definitely a tree.  
(II) The graph $G$ is definitely not a tree.  
(II) The graph $G$ may or may not be a tree.  

(a) $G$ has 7 vertices and 5 edges. ( )  
(b) $G$ has 7 vertices and 6 edges. ( )  
(c) $G$ has 7 vertices and 7 edges. ( )  
(d) $G$ has 7 vertices and there is a path from every vertex to every other vertex. ( )  
(e) $G$ has 7 vertices and every edge of $G$ is a bridge. ( )  
(f) $G$ has 7 vertices and no circuit. ( )  
(g) $G$ has 7 vertices and an Euler circuit. ( )
3. Let graph given as below. Please answer the following questions.  

(a) What is the degree of vertex D? (2 points) 

(b) Is the graph a tree? (3 points) 

(c) How many paths from A to D? (2 points) 

(d) How many odd vertices does this graph have? (2 points) 

(e) In order to get a minimal spanning trees, how many edges need to be removed? (3 points) 

(f) Does this graph have any Hamilton circuit? If, yes, please list them all with starting point A. If no, please explain why. (4 points) 

(g) Determine whether the graph has an Euler path, an Euler circuit or neither. Explain your answers with Euler’s theorems. (6 points)
4. Given the weighted graph as below.

(a) Apply the nearest-neighbor algorithm to find a Hamilton circuit with starting vertex A. (5 points) 

(b) Write the circuit as it would start with vertex C. (3 points) 

(c) Apply the cheapest-link algorithm to find a Hamilton circuit. (5 points) 

(d) Apply the Kruskal’s algorithm to get a minimal spanning tree for this graph. Please draw your minimal spanning tree below. (5 point)

5. Given digraph as below(2 points each).

(a) What is the indegree of vertex A? 

(b) What is the outdegree of vertex A? 

(c) Find all vertices that are incident to E. 

(d) Find all arc adjacent to DE. 

(e) Find a cycle in the digraph.
6. Use the following project digraph

(a) List the decreasing time list of the tasks from A to G.  
(b) Schedule the project with two processors using the decreasing time algorithm. Please show your work to get full credit.  

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START(0)  →  B(5)  →  D(12)  →  F(1)  
    ↓           ↓           ↓           ↓
A(8)  →  C(9)  →  E(6)  →  G(2)  →  END(0)
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10 pt