Section 3.1
5. (f) The limit doesn’t exist because \( \lim_{n \to \infty} \cos(2n\pi) = 1 \) and \( \lim_{n \to \infty}((2n+1)\pi) = -1 \), and the two sequences \((2n\pi)\) and \(((2n+1)\pi)\) both converge to \( \infty \).

8. (e) The asymptotes are \( y = 0 \) and \( y = x^3 \).

16. First, solve the equation for \( y \):
   \[
y = k \pm b \sqrt{\frac{(x-h)^2}{a^2} - 1}.
   \]

There are four situations to consider, based on using the plus or minus square root and sending \( x \) to \( \infty \) or \( -\infty \). Here’s the whole story for the plus square root and \( x \to \infty \):
   \[
   \lim_{x \to \infty} \left( \sqrt{\frac{(x-h)^2}{a^2} - 1} - \frac{x-h}{a} \right) = \lim_{x \to \infty} \frac{1}{\sqrt{\frac{(x-h)^2}{a^2} - 1 + \frac{x-h}{a}}} = 0.
   \]

Note that
   \[
   \lim_{x \to \infty} \sqrt{\frac{(x-h)^2}{a^2} - 1} = \infty
   \]

because \((x-h)^2/a^2 - 1 \geq \frac{1}{2} (x-h)/a \) if \( x \geq h + a\sqrt{2} \).

Section 3.2
1. (d) The limit is zero. Given \( \varepsilon > 0 \), choose \( \delta = \delta \). If \( 0 < |x| < \delta \), then
   \[
   \left| \frac{x^2}{|x|} - 0 \right| = |x| < \delta = \varepsilon.
   \]

8. (a) The limit is 1 because \( f(x) = 1 \) if \( |x-3/8| < 1/24 \).

10. Given \( M > 0 \), take \( \varepsilon = 1/M \). Then there is \( \delta > 0 \) such that \( |f(x)| < \varepsilon \) if \( 0 < |x-a| < \delta \).

Then \( 0 < |x-a| < \delta \) implies that
   \[
   \frac{1}{|f(x)|} > \frac{1}{\varepsilon} = M.
   \]

Section 3.3
10. (a) By Theorem 3.3.7,
   \[
   \lim_{x \to \infty} \sin \frac{1}{x} = \lim_{t \to 0^+} \sin t = 0.
   \]

(b) There’s nothing to do here.

(c) The roots are \( x = 1/(n\pi) \) for \( n \in \mathbb{N} \), and the values of \( x \) for which \( \sin \frac{1}{x} = 1 \) are...
2

\[ x = 2/(\pi) \). The largest such value is 2/\pi and the largest root is 1/\pi. The function \sin \frac{1}{x} is an odd function.

11. (b) False, the function

\[ f(x) = \begin{cases} 1 & \text{if } x > 0, \\ 0 & \text{if } x < 0 \end{cases} \]

has \( \lim_{x \to \infty} f(x) = 1 \), but \( \lim_{x \to 0} f(1/x) \) doesn’t exist.

16. (a) This chain of equalities is correct.
(b) This chain of inequalities is not correct because \( x^{1/2} \) is undefined if \( x < 0 \).

Section 3.4

21. True because, if \( \lim_{x \to 0^+} f(x) = 0 \), then for any \( \varepsilon > 0 \), there is \( \delta > 0 \) such that \( |f(x)| < \varepsilon \) if \( 0 < x < \delta \). But if \( 0 < -x < \delta \), then \( |f(-x)| = |f(x)| < \varepsilon \), so \( \lim_{x \to 0^-} = 0 \), too.

Section 3.5

5. (a) Take \( a = 0, b = 1, \) and \( f(x) = 1/x \).
(b) By Theorem 3.5.1 and Problem 3.5.3, the limits exist and they are finite. Set \( \theta = (d-c)/2 \). If \( c < t < c + \theta \) and \( d - \theta < y < d \), then \( t < y \), so \( f(t) \leq f(y) \). Now let \( \varepsilon > 0 \) be given and choose \( \delta_1 \) and \( \delta_2 \) so that

\[ |f(t) - \lim_{x \to c^+} f(x)| < \varepsilon/2, \quad |f(y) - \lim_{x \to d^-} f(x)| < \varepsilon/2 \]

if \( c < t < c + \delta_1 \) and \( d - \delta_2 < y < d \). Now take \( t = c + \min\{\delta_1, \theta\} \) and \( y = d - \min\{\delta_2, \theta\} \). Then \( f(t) \leq f(y) \), so

\[ \lim_{x \to c^+} f(x) < f(t) + \varepsilon/2 \leq f(y) + \varepsilon/2 < \lim_{x \to d^-} f(x) + \varepsilon. \]

Since this inequality between the limits is true for every \( \varepsilon > 0 \), it follows that

\[ \lim_{x \to c^+} f(x) \leq \lim_{x \to d^-} f(x). \]

Section 4.1

2. (b) This function is continuous everywhere it’s defined because the domain of \( f \) consists only of isolated points.
(h) This function is continuous for \( x \neq 0 \). The point \((0, 0)\) is not a singleton.

3. (a) Both functions are discontinuous at 0 because the limits are different along the two subsequences \( x_n = 1/n \) and \( x_n = 1/(n\pi) \). (For both functions, the limit along the first sequence is 0 but the limit along the second sequence is 1.)
(b) Both functions are continuous at 1/2 because \( f(1/2) = g(1/2) = 1/2 \) and both functions have a limit of 1/2 as \( x \) approaches 1/2.

5. (a) False. Take \( a = 0, b = 1, \) and

\[ f(x) = \begin{cases} \frac{1}{x} & \text{if } x > 0, \\ 0 & \text{if } x = 0. \end{cases} \]
(b) False. Take \(a = 0, \ b = 1,\) and \(f(x) = 1/2.\)

Section 4.2
1. (a) \(x = 0\) is not a point of removable discontinuity because
\[
\lim_{x \to 0^+} f(x) = 1, \ \lim_{x \to 0^-} f(x) = 1.
\]
(b) \(x = 0\) is a point of removable discontinuity because
\[
\lim_{x \to 0} \frac{\sin x}{x} = 1.
\]

2. (d) The set of points of discontinuity is exactly the set of all natural numbers. All the discontinuities are jump discontinuities.
(e) The set of points of discontinuity is exactly the set of rational numbers. (Note the function is not defined at \(x = 0.\)) The discontinuities are all removable discontinuities because \(\lim_{x \to a} f(x) = 0\) for all \(a \in \mathbb{R}.\)

Section 4.3
1. (a) Example 1: The function \(f(x) = 1/x\) is continuous on the bounded but not closed interval \((0, 1).\)
Example 2: The function \(f(x) = x\) is continuous on the unbounded, closed interval \((0, \infty).\)
Example 3: The function
\[
f(x) = \begin{cases} \frac{1}{x} & \text{if } x > 0, \\ 0 & \text{if } x = 0 \end{cases}
\]
is discontinuous on the closed, bounded interval \([0, 1].\)

4. The function \(f(x) = x^2 + 1\) has no real roots.