HOMEWORK #1 SOLUTIONS

Section 1.1
1. (b) \( A \cup (B \cap C) = \{1, 3, 5\} \).
   (d) \((A \cap B) \times C = \{(1,1), (1,8)\} \).
   (f) \(\emptyset \cap A = \emptyset \).

4. (b) If \( x \in A \cap B \), then \( x \in A \) and \( x \in B \), so \( x \notin A \setminus B \), so \( x \notin A \setminus (A \setminus B) \). If \( x \notin A \setminus B \) and \( x \in B \), and therefore \( x \in A \cap B \).
   (e) By part (b), if \( x \in A \cap B \), then \( x \notin A \setminus B \). Hence \( A \cap B \) and \( A \setminus B \) are disjoint.
   To show that \( A = (A \cap B) \cup (A \setminus B) \), we show first that \( A \subset (A \cap B) \cup (A \setminus B) \). To this end, let \( x \in A \). There are two cases: (i) if \( x \in B \), then \( x \in A \cap B \); (ii) if \( x \notin B \), then \( x \in A \setminus B \).

Section 1.2
3. (a) This is a bijection. If \( f(x_1) = f(x_2) \), then \( 2x_1 - 1 = 2x_2 - 1 \), so \( x_1 = x_2 \), and therefore \( f \) is injective. If \( y \in \mathbb{R} \), then \( y = f((x+1)/2) \), so \( f \) is surjective.
   (b) This is an injection because if
   \[
   \frac{x^2 - 1}{x - 1} = \frac{y^2 - 1}{y - 1},
   \]
   then \( x + 1 = y + 1 \), so \( x = y \). It isn’t surjective because there is no \( x \) with \( f(x) = 2 \).
   (c) This is a bijection. If \( \sqrt{x} = \sqrt{y} \), then \( x = y \), so \( f \) is injective. For \( y \in \mathbb{R} \), \( y = f(y^3) \), so \( f \) is surjective.
   (d) This is neither injective nor surjective: \( f(1) = f(-1) \), and there is no \( x \in [-1, 1] \) such that \( f(x) = 9/4 \).

Section 1.3
2. (a) The statement \( P(1) \) is “\( 1^2 + 1 \) is even”, which is true because \( 1^2 + 1 = 2 \).
   Now suppose \( k^2 + k \) is even. Then \( (k+1)^2 + (k+1) = k^2 + 2k + 1 + k + 1 = (k^2 + k) + 2(k+1) \).
   By the induction hypothesis, \( k^2 + k \) is even, and \( 2k + 2 \) is clearly even, so \( (k+1)^2 + (k+1) \) is a sum of two even numbers and therefore it’s even.