SECOND IN-CLASS EXAM

NO CALCULATORS! CLOSED BOOK! SHOW ALL WORK!

1. State and solve the steady state problem (5 points) and state the transient problem (10 points) for the initial boundary value problem

\[ u_t = u_{xx} + 1 - u \quad 0 < t, \quad 0 < x < 1, \]

\[ u(0, t) = 1, \quad u_x(1, t) = 0 \quad 0 < t, \]

\[ u(x, 0) = x \quad 0 < x < 1. \]

2. (25 points) by using the method of separation of variables, it can be shown (but you don’t have to) that the initial boundary value problem

\[ u_t(x, t) = 2u_{xx} \quad 0 < t, \quad 0 < x, \]

\[ u(0, t) = 0 \quad 0 < t, \]

\[ u(x, 0) = \begin{cases} 
1 & 0 < x < \pi, \\
0 & \pi < x 
\end{cases} \]

has the solution

\[ u(x, t) = \int_0^\infty B(\lambda) \sin \lambda x \exp(-2\lambda^2 t) \, d\lambda. \]

Find the coefficient function \( B(\lambda). \)

3. (25 points) State the eigenvalue problem for the differential equation plus boundary conditions

\[ u_t = 6u_{xx} \quad 0 < t, \quad 0 < x < \pi, \]

\[ u_x(0, t) = 0, \quad u_x(\pi, t) + u(\pi, t) = 0 \quad 0 < t. \]

4. (25 points) solve the eigenvalue problem (that is, find all eigenvalues and eigenfunctions, and show all steps):

\[ X''(x) = \sigma X(x), \quad 0 < x < \pi, \quad X(0) = 0, \quad X(\pi) = 0. \]

5. (10 points) Problems 2, 3, 4 correspond to the steps in solving an initial boundary value problem. Put them in the correct order. (No partial credit.)