FIRST IN-CLASS EXAM
NO CALCULATORS! CLOSED BOOK! SHOW ALL WORK!

1. (31 points) Write the Fourier series for the function
   \[ f(x) = x^2 + x, \quad -1 < x < 1. \]

2. (12 points each) (a) Sketch the function that the Fourier series of
   \[ f(x) = \begin{cases} 
   x^2 & 0 < x < \pi \\
   1 & x = 0 \\
   \sin x & -\pi < x < 0 
\end{cases} \]
   converges to.
   (b) Sketch the function that the Fourier sine series of
   \[ f(x) = x^2, \quad 0 < x < \pi \]
   converges to.

3. (20 points) Find the Fourier transform (complex Fourier integral coefficient) of the function
   \[ f(x) = \begin{cases} 
   e^{-x} & x > 0 \\
   0 & x \leq 0. 
\end{cases} \]

4. (15 points) Given that
   \[ |\sin x| = \frac{1}{\pi} - \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{1}{4n^2 - 1} \cos 2nx, \]
   show that
   \[ \frac{1}{2} = \frac{1}{3} + \frac{1}{15} + \frac{1}{35} + \ldots. \]

5. (10 points) The function \( f(x) = \ln(x + 1) \) for \( 0 < x < \pi \) Use the following table
to give an arithmetic expression for \( \hat{a}_2 \), the numerical approximation to the second Fourier cosine coefficient of \( f \).

<table>
<thead>
<tr>
<th>( i )</th>
<th>( x_i )</th>
<th>( \cos x_i )</th>
<th>( \cos(2x_i) )</th>
<th>( \cos(3x_i) )</th>
<th>( f(x_i) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1 ( \frac{\pi}{3} )</td>
<td>.5</td>
<td>- .5</td>
<td>-1</td>
<td>.71647</td>
<td></td>
</tr>
<tr>
<td>2 ( \frac{2\pi}{3} )</td>
<td>-.5</td>
<td>.5</td>
<td>1</td>
<td>1.12959</td>
<td></td>
</tr>
<tr>
<td>3 ( \pi )</td>
<td>-1</td>
<td>1</td>
<td>-1</td>
<td>1.42108</td>
<td></td>
</tr>
</tbody>
</table>