PRACTICE FIRST IN-CLASS EXAM

Directions: To receive full credit, you must show all work. You may use a calculator to do the arithmetic, but you must show all steps in the calculations. This exam was given during a 75 minute class session, so the actual exam may be shorter.

1. (25 points) Solve the system of linear equations

\[ x_1 - 7x_2 + x_5 = 3, \]
\[ x_3 - 2x_5 = 2, \]
\[ x_4 + x_5 = 1 \]

using Gauss-Jordan elimination.

2. (25 points) Compute the matrix product

\[ \begin{bmatrix} 0 & 0 & 1 \\ d & e & f \end{bmatrix} \begin{bmatrix} a & b & c \\ g & h & k \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}. \]

3. (15 points) Is the vector

\[ \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix} \]

a linear combination of the vectors

\[ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \text{ and } \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}. \]

(Recall that \( \vec{w} \) is a linear combination of \( \vec{v}_1 \) and \( \vec{v}_2 \) if there are numbers \( c_1 \) and \( c_2 \) so that \( \vec{w} = c_1 \vec{v}_1 + c_2 \vec{v}_2 \).)

4. (25 points) Decide whether the matrix

\[ \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 6 \end{bmatrix} \]

is invertible. If it is, find the inverse.

5. (10 points) Find a nonzero \( 2 \times 2 \) matrix \( B \) that commutes with the matrix

\[ \begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix}. \]