Math 307, Section E
Professor Lieberman
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SOLUTIONS TO FIRST IN-CLASS EXAM

1. First, we write the corresponding augmented matrix:

$$\begin{bmatrix} 2 & -1 & 4 & 3 \\ 1 & 2 & -3 & 2 \\ 4 & 3 & -2 & 1 \end{bmatrix}$$

and then we perform Gauss-Jordan elimination.

$$\begin{bmatrix} 2 & -1 & 4 & 3 \\ 1 & 2 & -3 & 2 \\ 4 & 3 & -2 & 1 \end{bmatrix} \begin{matrix} \div 2 \\ \\ \\ \end{matrix} \begin{bmatrix} 1 & -\frac{1}{2} & 2 & \frac{3}{2} \\ 1 & 2 & -3 & 2 \\ 4 & 3 & -2 & 1 \end{bmatrix} \begin{matrix} \\ -(I) \\ -4(I) \end{matrix} \begin{bmatrix} 1 & -\frac{1}{2} & 2 & \frac{3}{2} \\ 0 & \frac{5}{2} & -5 & \frac{1}{2} \\ 0 & 5 & -10 & -5 \end{bmatrix} \begin{matrix} \\ \div \frac{5}{2} \\ \\ \end{matrix}$$

$$\begin{bmatrix} 1 & -\frac{1}{2} & 2 & \frac{3}{2} \\ 0 & 1 & -2 & \frac{1}{5} \\ 0 & 5 & -10 & -5 \end{bmatrix} \begin{matrix} +\frac{1}{2}(II) \\ \\ -5(II) \end{matrix} \begin{bmatrix} 1 & 0 & 1 & 4 \\ 0 & 1 & -2 & \frac{1}{5} \\ 0 & 0 & 0 & -6 \end{bmatrix} \begin{matrix} \\ \\ \div -6 \end{matrix} \begin{bmatrix} 1 & 0 & 1 & 4 \\ 0 & 1 & -2 & \frac{1}{5} \\ 0 & 0 & 0 & -1 \end{bmatrix} \begin{matrix} -4(III) \\ -\frac{1}{5}(III) \\ \\ \end{matrix} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

(The last two steps are not critical for solving the problem, so skipping them will not count too many points.) Since the last row represents the equation $0 = 1$, this system has NO SOLUTION.

2. Again, we perform Gauss-Jordan elimination on the augmented matrix, but this time the right-hand side should start as the identity matrix I_3:

$$\begin{bmatrix} -1 & 2 & -3 & 1 & 0 & 0 \\ 2 & 1 & 0 & 0 & 1 & 0 \\ 4 & -2 & 5 & 0 & 0 & 1 \end{bmatrix} \begin{matrix} \div -1 \\ \\ \\ \end{matrix} \begin{bmatrix} 1 & -2 & 3 & -1 & 0 & 0 \\ 2 & 1 & 0 & 0 & 1 & 0 \\ 4 & -2 & 5 & 0 & 0 & 1 \end{bmatrix} \begin{matrix} \\ -2(I) \\ -4(I) \end{matrix}$$

$$\begin{bmatrix} 1 & -2 & 3 & -1 & 0 & 0 \\ 0 & 5 & -6 & 2 & 1 & 0 \\ 0 & 6 & -7 & 4 & 0 & 1 \end{bmatrix} \begin{matrix} \\ \div 5 \\ \\ \end{matrix} \begin{bmatrix} 1 & -2 & 3 & -1 & 0 & 0 \\ 0 & 1 & -\frac{6}{5} & \frac{2}{5} & \frac{1}{5} & 0 \\ 0 & 6 & -7 & 4 & 0 & 1 \end{bmatrix} \begin{matrix} +2(II) \\ \\ -6(II) \end{matrix}$$

$$\begin{bmatrix} 1 & 0 & \frac{3}{5} & -\frac{1}{5} & \frac{2}{5} & 0 \\ 0 & 1 & -\frac{6}{5} & \frac{2}{5} & \frac{1}{5} & 0 \\ 0 & 0 & \frac{1}{5} & \frac{8}{5} & -\frac{6}{5} & 1 \end{bmatrix} \begin{matrix} \\ \\ \div \frac{1}{5} \end{matrix} \begin{bmatrix} 1 & 0 & \frac{3}{5} & -\frac{1}{5} & \frac{2}{5} & 0 \\ 0 & 1 & -\frac{6}{5} & \frac{2}{5} & \frac{1}{5} & 0 \\ 0 & 0 & 1 & 8 & -6 & 5 \end{bmatrix} \begin{matrix} -\frac{3}{5}(III) \\ +\frac{6}{5}(III) \\ \\ \end{matrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & -5 & 4 & -3 \\ 0 & 1 & 0 & 10 & -7 & 6 \\ 0 & 0 & 1 & 8 & -6 & 5 \end{bmatrix}$$

Therefore, the original matrix is invertible and its inverse is

$$\begin{bmatrix} -5 & 4 & -3 \\ 10 & -7 & 6 \\ 8 & -6 & 5 \end{bmatrix}$$

3. In order to be a rotation (not a rotation-dilation), the matrix must have the form

$$\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}.$$

This means that you need $(1/2)^2 + (1/2)^2 = 1$, which isn't true, so the matrix DOES NOT represent a rotation. I will announce in class the grading policy about recognizing that the matrix does represent a rotation-dilation.

4. Just multiply out.

$$\begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix}^2 = \begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2k \\ 0 & 1 \end{bmatrix}$$

and

$$\begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix}^3 = \begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix}^2 \begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3k \\ 0 & 1 \end{bmatrix}.$$