Math 307, Section C2
Professor Lieberman
March 6, 2001

SECOND IN-CLASS EXAM

Directions: To receive full credit, you must show all work. You may use a calculator to do the arithmetic, but you must show all steps in the calculations.

1. (25 points) Find an orthonormal basis for
$$\text{span}\left\{ \begin{bmatrix} 2 \\ 3 \\ 0 \\ 6 \end{bmatrix}, \begin{bmatrix} 4 \\ 4 \\ 2 \\ 13 \end{bmatrix} \right\}.$$

2. (20 points) Consider two subspaces V and W of $\mathbb{R}^n$. Let $V + W$ be the set of all vectors in $\mathbb{R}^n$ of the form $\vec{v} + \vec{w}$, where $\vec{v}$ is in V and $\vec{w}$ is in W. Is $V + W$ necessarily a subspace of $\mathbb{R}^n$?

3. (30 points) Let A be the matrix
$$A = \begin{bmatrix} 1 & -1 & -1 & 1 & 1 \\ -1 & 1 & 0 & -2 & 2 \\ 1 & -1 & -2 & 0 & 3 \end{bmatrix}.$$
Find a basis for the kernel of A, a basis for the image of A and determine their dimensions.

4. (15 points) If A is an orthogonal $n \times n$ matrix, is A^2 necessarily orthogonal? If the answer is yes, explain why. If the answer is no, give an example.

5. (10 points) Is there a 3×3 matrix B so that $\text{im}(B) = \ker(B)$? Explain.

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THIRD IN-CLASS EXAM

Directions: To receive full credit, you must show all work. (For problem 2(b), you do not need to calculate the eigenvalues.) You may use a calculator to do the arithmetic, but you must show all steps in the calculations.

1. (30 points) Compute the determinant of the matrix

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 2 \\ 1 & 1 & 3 & 3 \\ 1 & 1 & 1 & 4 \end{bmatrix}.$$

2. (a) (15 points) Find all (complex) eigenvalues for the matrix

$$\begin{bmatrix} 1 & -1 & 1 & -1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}.$$

(b) (15 points) Given that the matrix

$$A = \begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}$$

has eigenvalues $\lambda = -2$ and $\lambda = 4$ (and no other eigenvalues), find a basis for the eigenspaces of A. Is there an eigenbasis?

3. (a) (15 points) Determine the characteristic polynomial of

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ a & b & c \end{bmatrix}$$

in terms of a, b, and c.
(b) (5 points) Use your answer to part (a) to find a 3×3 matrix with characteristic polynomial $\lambda^3 - 17\lambda^2 + 5\lambda - \pi$.

4. (10 points per part) An $n \times n$ matrix A is called *skew-symmetric* if $A^T = -A$.
(a) Give an example of a nonzero 2×2 skew-symmetric matrix.
(b) Show that a skew-symmetric 3×3 matrix A must have zero determinant. What does this say about the invertibility of A?