

PRACTICE FINAL EXAM

Find the general solutions of the equations in problems 1, 2, and 3.

1.  $x^2y'' + 2xy' - 6y = x$ .
2.  $y'' - 2y' - 3y = 3e^{2t}$ .
3.  $(2xy^2 + 2y) + (2x^2y + 2x)y' = 0$ .

Solve the initial value problems in problems 4 and 5.

4.  $y' + (\cot x)y = 2 \csc x$ ,  $y(\pi/2) = 1$ .
5.  $y'' + 4y' + 5y = 0$ ,  $y(0) = 1$ ,  $y'(0) = 0$ .
6. (a) Use Laplace transform methods to solve

$$y'' + 2y' + y = f(t), \quad y(0) = 1, \quad y'(0) = 0$$

where

$$f(t) = \begin{cases} 0 & \text{if } 0 \leq t < \pi/2, \\ \sin t & \text{if } t \geq \pi/2. \end{cases}$$

- (b) Find the Laplace transform of

$$te^{2t} \sin 3t$$

7. Use matrix methods to solve

$$\mathbf{x}' = \begin{pmatrix} 1 & 1 \\ 4 & -2 \end{pmatrix} \mathbf{x}.$$

8. Find the first five non-zero terms in a power series solution to

$$y'' - (x + 2)y = 0, \quad y(0) = 1, \quad y'(0) = 1.$$

9. (a) Determine the critical points of the system

$$\frac{dx}{dt} = (2 + x)(y - x), \quad \frac{dy}{dt} = (4 - x)(y + x).$$

- (b) Find the corresponding linear system for the critical point  $(4, 4)$ .
- (c) Determine the type and stability of the critical point  $(4, 4)$ .