SOLUTIONS TO PRACTICE SECOND IN-CLASS EXAM  
correction on problem 4 of the pencil and paper part

“Pencil & Paper” Part

1. \( y' = \sin x + x \cos x \)

2. \( y' = 2x \sec x + x^2 \sec x \tan x \)

3. \( y' = \frac{2x(1 - x) - (1 + x^2)}{(1 - x)^2} \)

4. \( y' = -\csc^2(x^2)2x \) (This was mistyped the first time.)

“Work-out” Part

1. Differentiation of the equation gives  
\[ \sqrt{1 + y} + x\frac{1}{2\sqrt{1+y}}y' + y'\sqrt{1+2x} + y\frac{1}{2\sqrt{1+2x}}2 = 2, \]
so  
\[ y' = \frac{2 - \sqrt{1+y} - \frac{y}{\sqrt{1+2x}}}{\frac{2}{2\sqrt{1+y}} + \sqrt{1+2x}}. \]

2. The function is \( f(x) = x^3 \) and \( a = 2 \). (Another acceptable answer is \( f(x) = (2 + x)^3 \) and \( a = 0 \).) Since \( f'(x) = 3x^2 \), the limit equals \( f'(2) = 12 \).

3. If the plane flies at an angle of \( \theta \) from the positive \( x \)-axis, then its velocity is \( \langle 600 \cos \theta, 600 \sin \theta \rangle \), and its velocity relative to the ground is \( \langle 600 \cos \theta, 600 \sin \theta \rangle + \langle 80 \cos \frac{7\pi}{4}, 80 \sin \frac{7\pi}{4} \rangle \).

This will be in the \( \mathbf{i} \) direction if the second component is zero:  
\[ 600 \sin \theta + 80 \sin \frac{7\pi}{4} = 0. \]
It follows that \( \sin \theta = -2 \sin(\pi/4)/15 = \sqrt{2}/15 \), which gives \( \theta = .09442 \) radians.
4. Differentiation yields

\[ f'(x) = (4x - 5)(2x^2 - 9x + 10) + (2x^2 - 5x + 3)(4x - 9), \]

so \( f'(2) = -1 \). An equation for the tangent line to the graph of \( f \) at the point \( x = 2 \) is

\[ y - 0 = -1(x - 2) \]

because \( f(2) = 0 \).

5. The velocity is

\[ s'(t) = \frac{1(t^2 + 9) - (t + 4)(2t)}{(t^2 + 9)^2} = \frac{-t^2 - 8t + 9}{(t^2 + 9)^2}. \]

The body is instantaneously at rest if \( s'(t) = 0 \) which gives \(-t^2 - 8t + 9 = 0\) so \( t = -9 \) or \( t = 1 \). It’s moving in the positive direction if \( s'(t) > 0 \) which means \(-9 < t < 1 \).

6. First, we use some algebra:

\[ \frac{\tan 1000x}{x} = \frac{\sin 1000x}{1000x} \frac{1000}{\cos 1000x}. \]

Since the limit of each term exists, the limit of the product exists and

\[ \lim_{x \to 0} \frac{\tan 1000x}{x} = \lim_{x \to 0} \frac{\sin 1000x}{1000x} \lim_{x \to 0} \frac{1000}{\cos 1000x} = 1000. \]