Edge $e = uv$ in a connected graph $G$ is a **bridge** if $G - e$ is disconnected.

Edge $e$ in a graph $G$ is a **bridge** if the number of connected components in $G - e$ is more than in $G$.

**Theorem 4.1** An edge $e$ of a graph $G$ is a bridge if and only if $e$ lies on no cycle of $G$.

A graph is **acyclic** if it has no cycles.

A graph $G$ is **tree** if $G$ is acyclic and connected.

1: List all non-isomorphic trees on 4 vertices

**End-vertex** or **leaf** is a vertex if degree one.

Tree is a **star** if it has exactly one vertex that are not a leaf.

Tree is a **double-star** if it has exactly two vertices that are not leaves.

Tree is $G$ a **caterpillar** if $G$ has at least 3 vertices and removing all leaves from $G$ gives a path, the path is called **spine** of the caterpillar.

Sometimes a tree $G$ has a vertex called **root**, then $G$ is **rooted tree**.

An acyclic graph is called a **forrest**.

**Theorem 4.2** A graph $G$ is a tree if and only if every two vertices of $G$ are connected by a unique path.

**Theorem 4.3** Every nontrivial tree has at least two end-vertices.

*Hint:* Take longest path.

**Theorem 4.4** Every tree of order $n$ has size $n - 1$. (recall order = $|V|$ and size = $|E|$)
2: 4.7 (a) Draw all forests of order 5. (b) Draw all trees of order 6.

3:  Show that if $T$ is a tree and $\Delta(T) = k$ then $T$ has at least $k$ leaves.

4: 4.9 Show that a graph of order $n$ and size $n - 1$ need not be a tree

5:  Show that sequence of natural numbers $d_1 \geq d_2 \geq \ldots \geq d_n \geq 1$ is a degree sequence of some tree iff $\sum d_i = 2n - 2$.

6:  Prove that every $n$ vertex graph with $m$ edges has at least $m - n + 1$ cycles (different cycles, but not necessarily disjoint cycles).

7:  Prove that a graph $G$ is a tree if and only if $G$ contains no cycle, but $G + uv$ does for each pair of non-adjacent vertices $u, v$ in $G$.

8:  Let $G$ be a connected graph that has neither $C_3$ nor $P_4$ as an induced subgraph. Prove that $G$ is a complete bipartite graph.

9:  *Open problem* In a 3-regular graph, is there always a cycle whose length is a power of 2? Is it true for the Petersen’s graph?

Reading for next time: Chapters 4.2, 4.3

from G. Chartrand and P. Zhang. “A First Course in Graph Theory”