Degree sequence of a graph is a sequence of it’s vertex degrees.

A finite sequence of nonnegative integers is graphical if it is a degree sequence of some graph.

**Theorem 2.10 Havel-Hakimi** A non-increasing sequence \( s : d_1, d_2, \ldots, d_n (n \geq 2) \) of non-negative integers, where \( d_1 \geq 1 \), is graphical if and only if the sequence

\[
s_1 : d_2 - 1, d_3 - 1, \ldots, d_{d_1+1} - 1, d_{d_1+2}, \ldots, d_n.
\]

is graphical.

**Proof:**

1: *Example 2.11* Decide whether the sequence \( s : \ldots \) is graphical.

How to store graph?

Let \( G = (V, E) \) be a graph, where \( V = \{v_1, v_2, \ldots, v_n\} \) and \( E = \{e_1, \ldots, e_m\} \).

**Adjacency matrix** of \( G \) is \( n \times n \) matrix \( A = [a_{ij}] \) where

\[
a_{i,j} = \begin{cases} 
1 & v_iv_j \in E \\
0 & \text{otherwise}
\end{cases}
\]

**Incidence matrix** of \( G \) is \( n \times m \) matrix \( B = [b_{ij}] \) where

\[
b_{i,j} = \begin{cases} 
1 & v_i \in e_j \\
0 & \text{otherwise}
\end{cases}
\]

**Theorem 2.13** \( A^k_{i,j} \) counts the number of \( v_i - v_j \) walks of length \( k \).
2: Find an example of two different graphs with the same degree sequence.

3: Is 5, 5, 3, 3, 2, 2, 2, 2, 2 graphical? Justify your answer.

4: 2.39 Let A be the adjacency matrix for $P_4$. Determine $A^4$ without computing A or performing matrix multiplication.

5: Let $G$ be a bipartite graph. Show that $A^k$ will have zero entries for each value of $k$.

6: An edge $e$ is a **bridge** if $G - e$ has more components than $G$. Show that if $G$ has no odd vertices, then $G$ has no bridges.

7: Prove that every graph with at least two vertices has at least two vertices of equal degree.

8: Show that $G \Box H$ is connected if and only if both $G$ and $H$ are connected. ($G \Box H$ denotes the Cartesian product of $G$ and $H$.)

9: **Open problem** In a connected graph, pick any three paths of maximum length. Is there always a vertex that lies on all of them?

Reading for next time: Chapters 3.1, 3.2

from G. Chartrand and P. Zhang. “A First Course in Graph Theory”