Path $P_n$ of length $n - 1$ has vertices $v_1, \ldots, v_n$ and edges $v_i v_{i+1}$ for all $1 \leq i \leq n - 1$.

Cycle $C_n$ of length $n$ if obtained from $P_n = v_1, \ldots, v_n$ by adding edge $v_1 v_n$.

Complete graph $K_n$ has $n$ vertices and for all $u, v \in V(K_n)$, $uv \in E(K_n)$, i.e. all edges.

**1:** What is $|E(K_n)|$?

The complement $\overline{G}$ of a graph $G$ is graph where $V(\overline{G}) = V(G)$ and $uv \in E(G)$ iff $uv \notin E(\overline{G})$.

Complement of complete graph is empty graph (or independent set).

**Theorem 1.11** If $G$ is disconnected then $\overline{G}$ is connected.

Graph $G$ is **bipartite** if $V(G) = X \cup Y$, where $G[X]$ and $G[Y]$ are empty graphs.

**Theorem 1.12** Graph $G$ is bipartite iff $G$ does not contain an odd cycle.

Complete bipartite graph $K_{m,n}$ is a bipartite graph with parts $|V_1| = m$ and $|V_2| = n$ and for all $u \in V_1$ and $v \in V_2$ we have $uv \in E(K_{m,n})$.

$K_{1,n}$ is called a star.
A graph $G$ is $k$-partite if $V(G)$ can be partitioned to $V_1, \ldots, V_k$, where $G[V_i]$ induces an empty graph.

A graph is complete $k$-partite graph if it is $k$-partite and maximizes the number of edges.

A join $G + H$ is a graph obtained from $G \cup H$ by adding all edges $uv$, where $u \in V(G)$ and $v \in V(H)$.

A cartesian product of $G$ and $H$, denoted by $G \square H$ has $V(G \square H) = V(G) \times V(H) = \{(u, v) : u \in V(G), v \in V(H)\}$ and $E(G \square H) = \{(u, v), (x, y) : u = x$ and $vy \in E(H)$ or $v = y$ and $ux \in E(G)\}$.

Note: different notation that in the book!

A cross product of $G$ and $H$, denoted by $G \times H$ has $V(G \times H) = V(G) \times V(H) = \{(u, v) : u \in V(G), v \in V(H)\}$ and $E(G \times H) = \{(u, v), (x, y) : ux \in E(G)$ and $vy \in E(H)\}$.

Note: different notation that in the book!

2: What is $G \otimes H$?

3: 1.21 Draw the graph $3P_4 \cup 2C_4 \cup K_4$.

4: 1.25 Let $G$ be a graph of order 5 or more. Prove that at most one of $G$ and $\overline{G}$ is bipartite.

5: 1.27 For the following pairs $G, H$ of graphs, draw $G + H, G \square H, G \times H$.

(a) $G = K_5$ and $H = K_2$;
(b) $G = \overline{K}_5$ and $H = \overline{K}_3$;
(c) $G = C_5$ and $H = K_1$.

6: Find graph on $n$ vertices that maximizes the number of edges but has no $K_3$ as a subgraph.

Multigraph is a graph where edges can have multiplicities (multiedges) and loops (edge $vv$).

Directed graph (or digraph) has edges as ordered pairs rather than sets of size two.

Oriented graph is a graph where edges are oriented (directed).

7: What is the difference between directed graph and oriented graph?

Hypergraph is a graph where edges are any subsets of vertices (not just size 2).

Reading for next time: Chapters 1.3, 1.4