1: Let $G$ be a $k$-connected graph and let $S$ be any set of $k$ vertices. Show that if a graph $H$ is obtained from $G$ by adding a new vertex $w$ and joining $w$ to the vertices of $S$, then $H$ is also $k$-connected.

2: Let $G$ be a $k$-connected graph of order $n \geq 2k$ and let $U$ and $W$ be two disjoint sets of $k$ vertices of $G$. Prove that there exist $k$ disjoint paths connecting $U$ and $W$.

3: Use Theorem 5.21 or 5.22 to show that $\kappa(G) = \lambda(G)$ when $G$ is 3-regular. (I’m asking you to reprove Theorem 5.20, but with a different proof. Copying the proof in the book and not using Theorems 5.21 or 5.22 is not an acceptable solution.)

4: Let $u$ and $v$ be two vertices in an oriented graph $G$. Describe an algorithm to find the maximum number of internally disjoint $u-v$ paths (paths starting at $u$ and ending in $v$). (Hint: Use network flows. How to make sure every vertex is used only in one path?)

5: Consider the network below with given capacity and flow values. (The edge label $f,u$ means flow-value $f$ and capacity $u$.) Find a sequence of augmenting paths and augment the flow to a maximum flow.

6: Let $(G, u, s, t)$ be a network, and let $\delta^+(X)$ and $\delta^+(Y)$ be minimum $s$-$t$-cuts in $(G, u)$. Show that $\delta^+(X \cap Y)$ and $\delta^+(X \cup Y)$ are also minimum $s$-$t$-cuts in $(G, u)$.