TIDES

In this project we are concerned with the depth of a natural channel for boats at the mouth of an east coast river. Here, the depth of the channel is affected by the ebb and flow of tides. We are provided with the following data. A cross-section of the channel is basically a "V" with straight sides. When the tides are at their absolute lowest point, the channel is 40 feet wide and 7 feet deep at the center.

Almost there are two high tides and two low tides each day. Each day, the high tides occur at twelve hour intervals and the low tides occur half-way between the high tides. There is a 20 foot difference on the shoreline between a high tide mark and the next low tide mark. These tides are caused primarily by the orbit of the moon around the earth and are called the semidiurnal tides.

The actual interval between high and high is 12.4 hours, caused by the fact the moon orbits the earth.

The sun has an effect on the tides as well, but much less of an effect because of its distance from the earth. However, when the earth moon and sun are all in a straight line (either with the moon between the earth and the sun or with the earth between the sun and the moon) the effect is to move the high water mark even higher up the shoreline. The highest high tides are called spring tides. The tides in which there is the least difference between low and high are called neap tides. These are caused when the sun, earth and moon form a right angle. There is a 40 foot difference in the shoreline between the highest high tide and the lowest high tide. There are two spring tides (and two neap tides) at uniform intervals every 28 days.

Your first job is to set up a mathematical model of the situation and determine a function which gives the water level on the shoreline as a function of time. Use a top-down structure which decomposes the problem into two smaller problems and then combine the results to obtain a solution to determine the function we need. The two sub-problems are labeled a.) and b.) below.

a.) Ignore the effect of the sun. That is assume that the sun does not exist!

i.) Sketch a graph which describes the water level on the shoreline as a function of time, which results from the semidiurnal tides alone. Be sure to clearly label and explain your coordinate system.

ii.) Develop an algebraic rule which fits this model.

b.) Next assume the semidiurnal tides do not exist.

i.) Sketch a graph which describes the water level on the shoreline as a function of time, which results from the effects of the sun and moon that create spring tides and neap tides alone. Be sure to clearly label and explain your coordinate system.
ii.) Develop an algebraic rule which fits this model.

c.) Now, develop a rule which describes the effects of the two types of tides acting together by using your results from parts a.) and b.).

d.) Graph this function.

e.) Answer the following questions relating to your model. You need to justify your answer.

Using your rule in part c.), when during the month is the water at its highest point on the shoreline?

Using your rule in part c.), when is the water rising fastest?

f.) The channel must be 8 feet deep at the center all day long to be open to boat traffic. How many days a month will the channel be open?