Refraction of Light by a Water Droplet and Rainbows

L. Hogben 10/95
cf. Larson 3.3.66; Steven Janke, "Somewhere within the Rainbow"

The boundary surface between water and air can both reflect and refract. When it reflects, the angle of incidence α equals the angle of reflection α. See Fig. 1. Notice that all angles are measured from the perpendicular to the surface.

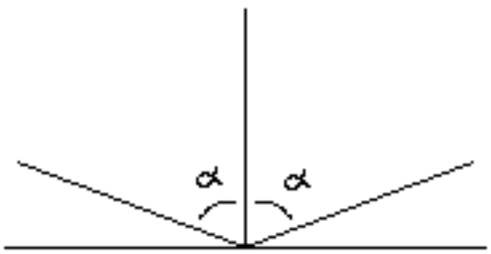

Figure 1

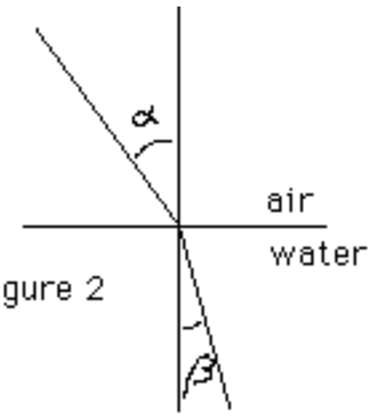

Figure 2

When light refracts it bends (see Fig. 2). The angle at which it bends depends on the two media (in our case, air and water). The relationship is governed by Snell's Law, $\frac{\sin \alpha}{\sin \beta} = k \approx 1.33$, where α is the angle with the perpendicular in air and β is the angle with the perpendicular in water.

When encountering a curved surface the light behaves as if the surface was the tangent line at that point. The behavior of a light ray incident on a drop of water that refracts, reflects, and refracts is show in Fig. 3. Figure 3 shows the cross-section of a drop of water.

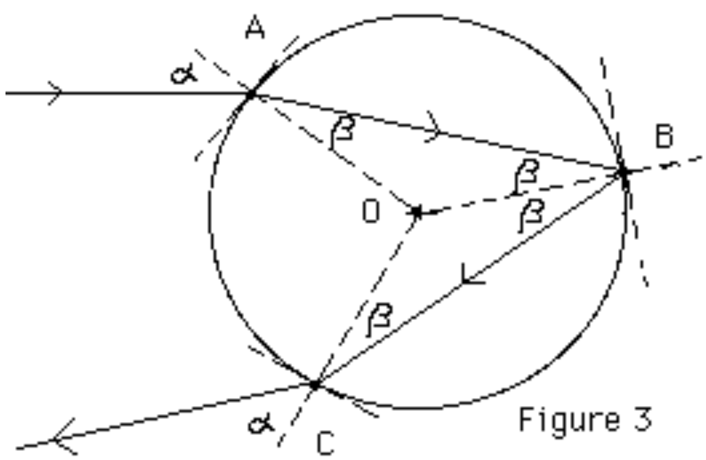

Figure 3

The light ray enters at A, refracting. It reflects at B and exits at C, refracting again. The behavior of the light at A and C is described by Snell's Law (i.e., if the angle in air is α then the angle in water is β). The behavior of the light at B under reflection requires the two angles at B to be equal.

1. Explain why the angle of refraction at A, β, must equal the angle of incidence at B, β, and the angle of reflection at B, β, must equal the angle of incidence at C, β.

The possibility illustrated in Figure 3 is not the only one. The light ray could reflect at A, or refract at A and reflect at B but these do not affect rainbow formation (reflection at A is unchanged and the other is on the wrong side). It is also possible to have more complicated routes, but as each reflection/refraction reduces intensity, these are less bright and we concentrate on the route just described.

We are interested in how much the ray is deflected when it leaves the drop. A ray entering on the diameter will have an angle of incidence of 0 at A, hence a refraction angle of 0 at A, and incidence and reflection angles of 0 at B, and thus incidence and refraction angles of 0 at C=A. Such a ray will have completely reversed direction and so will have an angle of deflection of π. In general, the angle of deflection will vary with the angle of incidence α.

2. Assume all light rays enter horizontally, parallel to the diameter, as shown in Fig. 3.
a) Show that the angle of deflection D is given by $D = \pi + 2\alpha - 4\beta$ (with angles measured in radians clockwise).
b) Use Snell's Law to express D as a function of α alone,

$$D(\alpha) = \pi + 2\alpha - 4 \arcsin\left(\frac{\sin \alpha}{k}\right).$$

3. Recall that $k \approx 1.33$ for water.
a) Sketch the graph of $D(\alpha)$ for $0 \leq \alpha \leq \frac{\pi}{2}$.
b) Find the angle α_{min} where the minimum occurs and the minimum value $D_{min} = D(\alpha_{min})$.
c) What is $\pi - D_{min}$ in degrees?

$\pi - D_{min}$ is called the rainbow angle. It is important because at the minimum there is very little change, so many rays behave the same way and the light is brighter. The rainbow angle is the angle between the incident beam of light (from the sun) and the light returning from the raindrop. There is a cone of rays with this angle about the incident ray direction, and a cross section of this cone perpendicular to its axis is a circle. See Fig. 4. Thus you might expect to see a bright circle of light for a rainbow. But part of it is below the horizon, so we see an arc (it is possible to see the whole circle from an airplane).

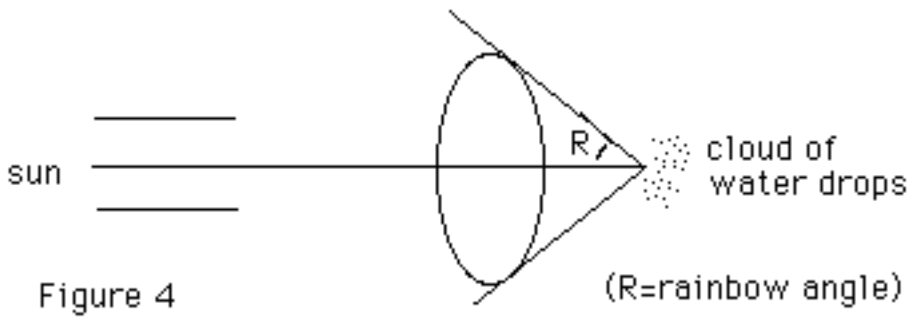

Figure 4

Different wavelengths of light have slightly different values of k, so the light separates out. Thus bright bands of color appear in a rainbow.

4. Prove that for a medium with refraction constant k the minimum deflection angle occurs when $\cos\alpha = \sqrt{\frac{k^2 - 1}{3}}$.

5. For visible light moving across an air-water boundary, the constant k in Snell's Law ranges from 1.3312 for blue light (wavelength 6563 angstroms) to 1.3372 for red light (wavelength 4861 angstroms).
a) Express the rainbow angle (in degrees) as a function of k, R(k).
b) Plot R(k) from 1.3312 to 1.3372 and determine the range of the rainbow angle for these values of k.