Math 165H Test 2 10/25/01 L. Hogben

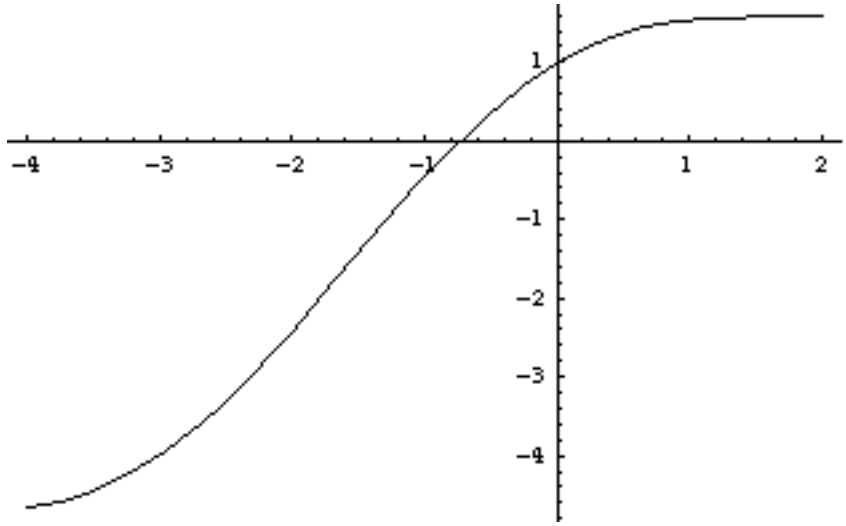
All answers must be justified by computation or explanation. Write your answers on your own paper (except #9). Show your work. Numbers such as $\sqrt{2}$ that do not have exact decimal representation should be left exact, except in Questions 7 and 8. All decimals should be given to 6 significant digits. All angles are in radians unless otherwise stated.

1. (10 points each) Find the derivative of the given function

   a) f(x) = $\dfrac{-7x^3+2x}{x^{-3}+2}$ b) f(x) = x $\cos\sqrt{\sin x+1}$

2. (10 points) g(x) = $\tan(\sqrt{x}-1)+4$. g′(1) = ?
3. (10 points) Find the equation of the line tangent to the graph of f(x) = ($5x^2$-3x+1) cos x at x = 0.
4. (10 points) Take the derivative of $7x^2$ - 3 from the definition.
5. (10 points) The position of a particle is given by **r**(t) = < t^3+1, $\dfrac{1}{t-1}$, $4\sqrt{t}$ > , where **r** is in cm and t is in seconds. Find the speed of the particle after 2 seconds.
6. (5 points) $\lim_{x\to 0}(3x)\csc(5x)$ = ?
7. (15 points) Euler's method can be used to approximate the values of an implicitly defined function. Carry out the first step of such an approximation with step size h = 0.1 starting at (1,2) for the implicit function (x^2+x)y - x y^2 = 0
8. (5 points each) The graph of the function f(x) = x + cos x is shown below.

   a) On the graph draw 2 iterations of Newton's method, starting at x = 0.



   b) Carry out enough iterations of Newton's method to find a root accurate to 0.000001. List all iterates.

9. (10 points) Gravel is being dumped from a conveyor belt at a constant rate onto a conical pile whose base diameter and height are always equal. When the height is 6 feet, the height is increasing at a rate of 1 ft/min. At what rate is the volume increasing?

6. $\lim_{x\to 0} 3x\csc(5x) = \lim_{x\to 0} \frac{3x}{\sin(5x)}$

$= 3 \lim_{x\to 0} \frac{1}{5} \frac{5x}{\sin(5x)} = \frac{3}{5} \lim_{5x\to 0} \frac{5x}{\sin(5x)} = \frac{3}{5}$

7. $(2x+1)y + (x^2+x)\frac{dy}{dx} - y^2 - 2yx\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{y^2 - (2x+1)y}{x^2+x-2xy}$

$x_0 = 1 \quad y_0 = 2 \quad h = 0.1$

$x_1 = x_0 + h = 1.1$

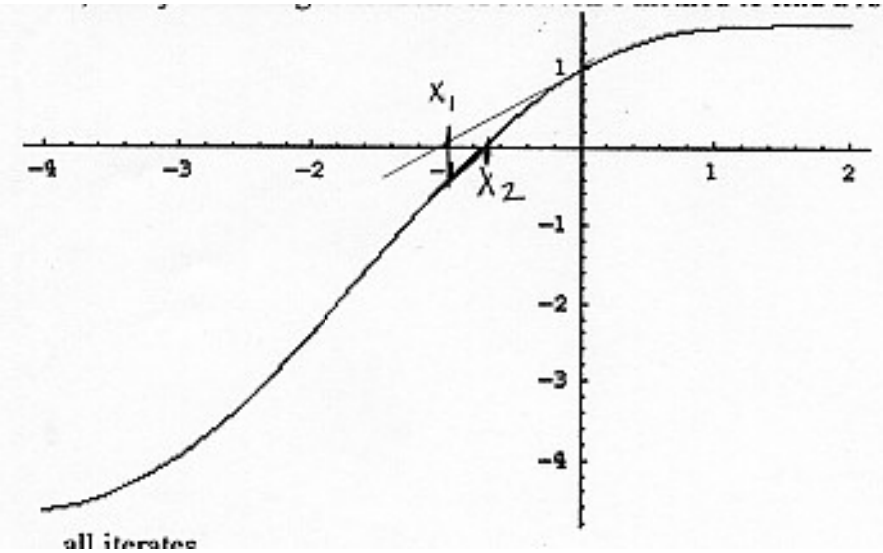
$y_1 = y_0 + h\left.\frac{dy}{dx}\right|_{\substack{x=1\\y=2}} = 2 + (0.1)\left(\frac{4-3\cdot 2}{1+1-4}\right) = 2+(0.1)\left(\frac{-2}{-2}\right)$

$= 2.1$

9. $V = \frac{\pi}{3}r^2h \qquad r = \frac{h}{2} \qquad V = \frac{\pi}{12}h^3$

$\frac{dV}{dt} = \frac{\pi}{12}(3h^2)\frac{dh}{dt} = \frac{\pi}{4}h^2\frac{dh}{dt}$

$\left.\frac{dV}{dt}\right|_{h=6} = \frac{\pi}{4}(6^2)\left(1\ \text{ft/min}\right) = 9\pi\ \text{ft}^3/\text{min}.$



all iterates

$X_0 = 0$

$X_1 = -1$

$X_2 = -0.750364$

$X_3 = -0.739113$

$X_4 = -0.739085$

$X_5 = -0.739085$

answer