Math 165H Test 1 9/20/01 L. Hogben

All answers must be justified by computation or explanation. Write your answers on your own paper. Show your work. All non-exact decimals should be given to 6 significant digits. All angles in radians unless otherwise stated. Any graph must either have its axes labeled or its window stated.

1. (10 points each) Find the limit or determine that it does not exist and prove your answer is correct.

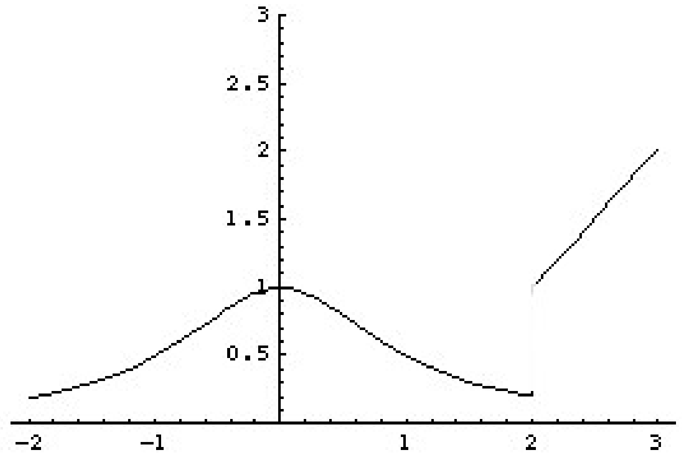
   A) $\lim_{x\to\infty}\frac{2x^2+1}{3x^2-5x}$ B) $\lim_{x\to 0}\left\langle \frac{x}{x+1}, \frac{1}{\cos x}\right\rangle$ C) $\lim_{x\to\pi}\sin x\cos\frac{1}{x-\pi}$

2. (part A is 10 points, parts B & C are 5 points each)
   Find the limit or determine that it does not exist.

   A) $\lim_{x\to 0}\frac{\tan x}{x}$

   B) $\lim_{x\to 0} g(x)$ for the function g whose graph is shown at right

   C) $\lim_{x\to 2} g(x)$ for the function g whose graph is shown at right



3. (5 points each) **v** = <2,-2,1>, **w** = <1,0,0>. Find

   A) **v** + 4 **w**

   B) angle between **v** and **w**

   C) proj $_{\mathbf{v}}$ **w**

   D) line through **w** in direction of **v**.

4. (10 points) Find $\lim_{x\to 2}(3.7x+0.1)$ and use the ε/δ-definition to prove your answer is correct.

5. (20 points) $f(x)=\begin{cases}\frac{x^2+4}{x+2} \text{ if } x<0\\ \sin(\pi x) \text{ if } 0\le x\le 2\\ \text{x - 2 if } 2<x\end{cases}$ For what real numbers x is f(x) discontinuous? Why?

1. A) $\lim\limits_{x\to\infty} \dfrac{2x^2+1}{3x^2-5x} = \lim\limits_{x\to\infty} \dfrac{\frac{1}{x^2}(2x^2+1)}{\frac{1}{x^2}(3x^2-5x)} = \lim\limits_{x\to\infty} \dfrac{2+\frac{1}{x^2}}{3-\frac{5}{x}} = \dfrac{2}{3}$

B) $\lim\limits_{x\to 0} \dfrac{x}{x+1} = \dfrac{0}{0+1} = 0 \quad \lim\limits_{x\to 0} \dfrac{1}{\cos x} = \dfrac{1}{\cos 0} = \dfrac{1}{1} = 1$

$\lim\limits_{x\to 0} \langle \dfrac{x}{x+1}, \dfrac{1}{\cos x} \rangle = \langle 0, 1 \rangle$

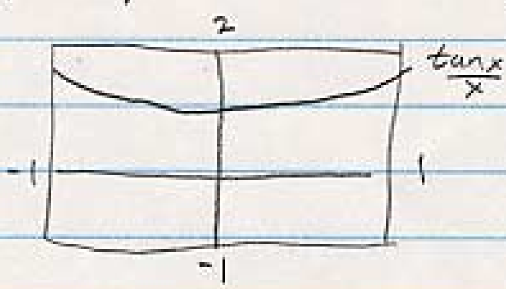
C) $|\sin x| \geq |\sin x \cos \frac{1}{x-\pi}| \quad -|\sin x| \leq -|\sin x \cos \frac{1}{x-\pi}|$

since $|\cos\theta| \leq 1$

$\lim\limits_{x\to\pi} |\sin x| = |\sin \pi| = 0 = \lim\limits_{x\to\pi} -|\sin x|$

So by Squeeze Thm $\lim\limits_{x\to\pi} \sin x \cos \frac{1}{x-\pi} = 0$

2. A)



$\lim\limits_{x\to 0} \dfrac{\tan x}{x} = 1$

B) $\lim\limits_{x\to 0} g(x) = 1$ because the function is continuous at $x=0$

C) $\lim\limits_{x\to 2} g(x)$ DNE because $\lim\limits_{x\to 2^-} g(x) \approx 0.2$ & $\lim\limits_{x\to 2^+} g(x) \approx 1$.

3. A) $\langle 2+4, -2+0, 1+0\rangle = \langle 6,-2,1\rangle$

B) $\cos\theta = \dfrac{v\cdot w}{\|v\|\|w\|} = \dfrac{2}{3\cdot 1} = \dfrac{2}{3}$ $\quad \theta = 0.841069$

C) $\text{proj}_{\vec v}\vec w = \dfrac{v\cdot w}{\|v\|}\dfrac{\vec v}{|v|} = \dfrac{2}{3}\cdot\dfrac{1}{3}\langle 2,-2,1\rangle = \langle \frac{4}{9}, \frac{-4}{9}, \frac{2}{9}\rangle$

D) $\vec r(t) = \langle 1,0,0\rangle + t\langle 2,-2,1\rangle$

4. $\lim\limits_{x\to 2}(3.7x+0.1) = (3.7)(2)+0.1 = 7.5$

Find δ:

$|3.7x+0.1-7.5| < \varepsilon$

$|3.7x-7.4| < \varepsilon$

$|x-2| < \frac{\varepsilon}{3.7}$

Choose $\delta = \frac{\varepsilon}{3.7}$

Prove δ works:

$|x-2| < \frac{\varepsilon}{3.7}$

$|3.7x-7.4| < \varepsilon$

$|3.7x+0.1-7.5| < \varepsilon$

5. Potential problem points: $x=-2$ (denominator 0), $x=0$, $x=2$ (definition splits here)

At $x=-2$: $\dfrac{x^2+4}{x+2} = \dfrac{4+4}{-2+2} = \dfrac{8}{0}$ limit $x\to -2$ DNE discontinuous

At $x=0$ $\lim\limits_{x\to 0-} f(x) = \lim\limits_{x\to 0-}\dfrac{x^2+4}{x+2} = 2$ $\quad \lim\limits_{x\to 0+} f(x) = \lim\limits_{x\to 0+}\sin\pi x = 0$

discontinuous

At $x=2$ $\lim\limits_{x\to 2-}\sin\pi x = \sin 2\pi = 0$ $\quad \lim\limits_{x\to 2} x-2 = 2-2 = 0$

So even though the definition splits at 2 f is continuous at $x=2$.

f is discontinuous at $x=-2$, $x=0$.