Title: Crankshaft design

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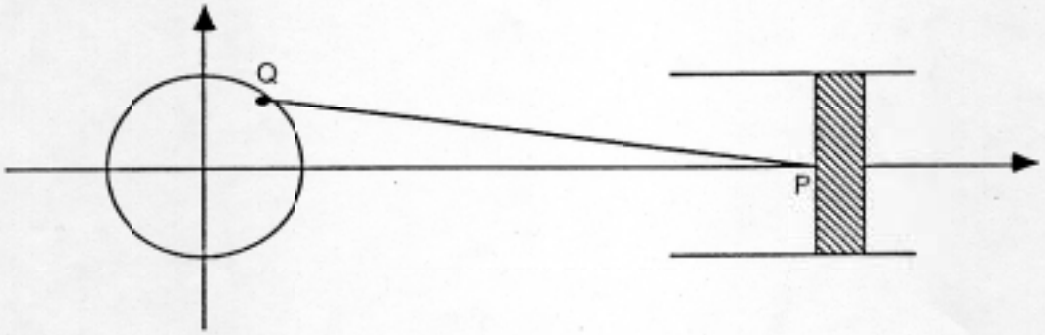
Problem statement:

In a reciprocating internal combustion engine, each piston is housed in a cylinder and attached to the rim of the crankshaft by a connecting rod as indicated in the figure below. The piston moves back and forth in the cylinder, and, in response, the crankshaft rotates counterclockwise.



Among the factors which determine the stress on certain engine parts are the speed and acceleration of the pistons. This seems plausible in the case of a connecting rod, for example, since force is proportional to acceleration and one of the main forces exerted on a connecting rod comes directly from its linkage to the piston. One common indication of this relationship between stress and piston motion is the warning "red line" found on tachometers is some sports and racing cars. A tachometer displays engine speed measured in revolutions per minute (rpm's) of the crankshaft. To push an engine past its "red line" rpm level is to risk serious damage due to excessive stress on pistons, connecting rods and the linkages between the connecting rods and the pistons and the crankshaft. In this problem you are asked to investigate various aspects of the relationship between crankshaft rpm's, piston speed and acceleration, connecting rod length and crankshaft radius.

In a certain automobile, suppose a 60 miles per hour cruising speed results from the crankshaft rotating at the constant rate of 3000 rpm's. If the radius of the crankshaft is 1 inch and the length of the connecting rod is 4 inches, find the piston's maximum and minimum velocity (in feet/second accurate to the nearest hundredth) and acceleration (in feet /sec/sec accurate to the nearest foot).

Is the piston motion in part (a) sinusoidal? That is, can the motion be described by a function of the form $x(t) = A+B\sin(Ct+D)$ for appropriately chosen constants A, B, C and D? [x(t) is the x coordinate of P in feet after t seconds.]

In designing an engine, a decision must be made as to how long the connecting rods should be. Is it better to have them as short as possible or longer? How much longer? One question that arises in this connection is how changing the connecting rod length would affect piston velocity and acceleration. Assuming the crankshaft's rotational velocity and radius hold constant at 3000 rpm's and 1 inch respectively, investigate the relationship between connecting rod length and maximum absolute value of piston velocity and acceleration. Do your results suggest any conclusions regarding ideal connecting rod length? What other factors seem likely to have an important bearing on the question of connecting rod length?

Another design question concerns the radius of the crankshaft and what's gained or lost as it changes size. Assuming the crankshaft speed is 3000 rpm's and the connecting rod length is 4 inches, find the maximum absolute value of piston velocity and acceleration if the crankshaft radius is doubled to 2 inches. If you assume the engine can generate approximately the same average piston speed regardless of the crankshaft radius, what trade-off does your result suggest is involved in making the crankshaft radius larger? That is, would the larger radius seem more appropriate for a dump truck or a race car? Why?